

# PERTURBATIONS OF HYPERCYCLIC VECTORS

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ABSTRACT. We show that a linear operator can have an orbit that comes within a bounded distance of every point, yet is not dense. We also prove that such an operator must be hypercyclic. This gives a more general form of the hypercyclicity criterion. We also show that a sufficiently small perturbation of a hypercyclic vector is still hypercyclic.

## 1. INTRODUCTION

For a separable complex Banach space  $X$ ,  $\mathcal{B}(X)$  will denote the algebra of all bounded linear operators on  $X$ . If  $T \in \mathcal{B}(X)$  and  $x \in X$ , then the orbit of  $x$  is  $\text{Orb}(T, x) = \{x, Tx, T^2x, \dots\}$ . An operator  $T$  is called hypercyclic if there is a vector  $x$  such that  $\text{Orb}(T, x)$  is dense. In this case,  $x$  is called a hypercyclic vector for  $T$ . It is surprising that a linear operator can actually be hypercyclic; the first example was constructed by Rolewicz [9] in 1969. He showed that if  $B$  is the backward shift on  $\ell^2(\mathbb{N})$ , then  $\lambda B$  is hypercyclic if and only if  $|\lambda| > 1$ . Since this first example a ‘‘Hypercyclicity Criterion’’ was developed independently by Kitai [8] and Gethner and Shapiro [6]. This criterion has been used to show that hypercyclic operators arise within the classes of composition operators [3], weighted shifts [10], adjoints of multiplication operators [7], and adjoints of subnormal and hyponormal operators [5]. This paper is motivated by questions about perturbations of hypercyclic vectors. The basic question is as follows:

*If  $\text{Orb}(T, x)$  is dense in  $X$  and  $\text{Orb}(T, y)$  is ‘‘small’’, then is  $\text{Orb}(T, x+y)$  dense in  $X$ ?*

It is easy to see that if  $\text{Orb}(T, y)$  is finite, or a convergent sequence, then the answer to this question is ‘‘yes’’. Here we address the question when  $\text{Orb}(T, y)$  is bounded or has compact closure. Notice that if  $\text{Orb}(T, y)$  is bounded, and  $\text{Orb}(T, x)$  is dense, then  $\text{Orb}(T, x+y)$  comes within a bounded distance of every point in  $X$ . More precisely, if  $\|T^n y\| \leq M$  for all  $n \geq 0$  and  $d > M$ , then  $\text{Orb}(T, x+y) \cap B(z, d) \neq \emptyset$  for all  $z \in X$ , where  $B(z, d)$  denotes the open ball centered at  $z$  with radius  $d$ . Hence, we want to study orbits of vectors that are  $d$ -dense, that is, orbits which intersect every open ball with radius  $d$ . The following question naturally arises:

*If an orbit is  $d$ -dense in  $X$ , then is it necessarily dense in  $X$ ?*

In this paper we show that if  $T$  has a  $d$ -dense orbit, then  $T$  must be hypercyclic. That is, some vector will have dense orbit under  $T$ . However, surprisingly, we show that a  $d$ -dense orbit need not be dense! In fact, we will show that for any  $\epsilon > 0$ , there are  $\epsilon$ -dense orbits that are not dense. We also construct a vector  $x$  with dense orbit and a vector  $y$  whose orbit has compact closure, and yet  $x+y$  does not

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have a dense orbit. However, we prove that if  $\text{Orb}(T, x)$  is dense and  $\text{clOrb}(T, y)$  is countable and compact, then  $\text{Orb}(T, x + y)$  is dense.

## 2. MAIN RESULTS

**Theorem 2.1.** *Suppose that  $X$  is a separable Banach space and  $T \in \mathcal{B}(X)$ . If  $T$  has a  $d$ -dense orbit for some  $d > 0$ , then  $T$  is hypercyclic.*

**Corollary 2.2.** *If  $T \in \mathcal{B}(X)$  and there is a  $d > 0$  such that for any open set  $U$  and any ball  $B(z, d)$  with radius  $d$ , there exists an  $n \geq 1$  such that  $T^n(U) \cap B(z, d) \neq \emptyset$ , then  $T$  is hypercyclic.*

*Proof of Corollary 2.2.* Suppose that  $T$  satisfies the given condition. For a given ball  $B(z, d)$  with radius  $d$ , our assumption says that  $\bigcup_n (T^n)^{-1}(B(z, d))$  is a dense open set. Thus, if  $\{z_k\}$  is a countable dense subset of  $X$ , then  $\bigcap_k \bigcup_n (T^n)^{-1}(B(z_k, d))$  is a dense  $G_\delta$  set in  $X$ . Furthermore, if  $x \in \bigcap_k \bigcup_n (T^n)^{-1}(B(z_k, d))$ , then  $\text{Orb}(T, x) \cap B(z_k, d) \neq \emptyset$  for all  $k$ . Thus, the orbit of  $x$  comes within a distance  $(d + \epsilon)$  of every point in  $X$ , (for any  $\epsilon > 0$ ) hence by Theorem 2.1,  $T$  is hypercyclic.  $\square$

We begin with a few Lemmas for the proof of Theorem 2.1.

**Lemma 2.3.** *If  $T \in \mathcal{B}(X)$  has a  $d$ -dense orbit for some  $d > 0$ , then for every  $\epsilon > 0$ ,  $T$  has an  $\epsilon$ -dense orbit.*

*Proof.* Suppose that  $x \in X$  and  $0 < d < \infty$  are such that for every  $y \in X$ , there exists an  $n \geq 0$  such that  $\|T^n x - y\| < d$ .

If  $\epsilon > 0$ , then set  $x_\epsilon = (\epsilon/d)x$ . If  $y \in X$ , then there exists an  $n \geq 0$  such that  $\|T^n x - \frac{d}{\epsilon}y\| < d$ . Thus,  $\|T^n x_\epsilon - y\| < \epsilon$ . Hence, the vector  $x_\epsilon$  has an orbit that is  $\epsilon$ -dense in  $X$ .  $\square$

A sequence  $(x_n)_{n=1}^\infty$  is called  $\epsilon$ -separated if  $\|x_n - x_k\| \geq \epsilon$  for all  $n \neq k$ . If  $X$  is any infinite dimensional normed linear space, then it's an easy exercise with the Hahn-Banach Theorem to show that  $X$  has a 1-separated sequence of unit vectors ([4, page 7]). In fact, it is a deep theorem due to J. Elton and E. Odell that there is an  $\epsilon > 0$  and a sequence of unit vectors in  $X$  that are  $(1 + \epsilon)$ -separated ([4, page 241]).

**Lemma 2.4.** *If  $\text{Orb}(T, x) \cap B(y, d) \neq \emptyset$  for every  $y \in X$ , then  $\text{Orb}(T, x) \cap B(y, 3d)$  is infinite for every  $y \in X$ .*

*Proof.* Suppose that  $\text{Orb}(T, x) \cap B(y, d) \neq \emptyset$  for every  $y \in X$ . Fix a  $y \in X$ ; then we will show that  $\text{Orb}(T, x) \cap B(y, 3d)$  is infinite. Since  $X$  is infinite dimensional, there exists an infinite sequence  $\{y_n\}$  in  $\partial B(y, 2d)$  that is  $2d$ -separated. Thus,  $B(y_n, d) \cap B(y_k, d) = \emptyset$  for  $n \neq k$ . Thus since  $B(y_n, d) \subseteq B(y, 3d)$  and the orbit of  $x$  must enter the ball  $B(y_n, d)$  for every  $n$ , then  $B(y, 3d)$  contains an infinite number of elements of the orbit of  $x$ .  $\square$

*Proof of Theorem 2.1.* We will show that  $T$  is transitive. So, suppose that  $U, V$  are any two nonempty open sets in  $X$ . Choose  $y \in U$  and  $z \in V$  and an  $\epsilon > 0$  such that  $B(y, 3\epsilon) \subseteq U$  and  $B(z, 3\epsilon) \subseteq V$ . Then for this choice of  $\epsilon$ , by Lemma 2.3, there exists a vector  $x_\epsilon$  whose orbit is  $\epsilon$ -dense in  $X$ . By the previous lemma the orbit of  $x_\epsilon$  intersects each of the balls  $B(y, 3\epsilon)$  and  $B(z, 3\epsilon)$  infinitely many times. Thus, there exists an  $n \geq 0$  such that  $T^n x_\epsilon \in U$  and a  $m > n$  such that  $T^m x_\epsilon \in V$ . Thus, if  $k = m - n$ , then  $T^k(U) \cap V \neq \emptyset$ . Hence  $T$  is hypercyclic.  $\square$

Recall that a vector  $x$  is a supercyclic vector for  $T$  if  $\{cT^n x : n \geq 0, c \in \mathbb{C}\}$  is dense. We now show that if the orbit of  $x$  comes within a bounded distance of every point, then  $x$  must be a supercyclic vector.

**Proposition 2.5.** *If  $T \in \mathcal{B}(X)$  and  $\text{Orb}(T, x)$  is  $d$ -dense in  $X$  for some  $d > 0$ , then  $x$  is a supercyclic vector for  $T$ .*

*Proof.* Let  $y \in X$  and  $\epsilon > 0$ . Choose an  $n > 0$  such that  $\frac{d}{n} < \epsilon$ . Then there exists a  $k \geq 0$  such that  $\|T^k x - ny\| < d$ , hence it follows that  $\|\frac{1}{n}T^k x - y\| < d/n < \epsilon$ . Hence  $x$  is a supercyclic vector for  $T$ .  $\square$

Thus we have shown that if  $\text{Orb}(T, x)$  is  $d$ -dense, then  $T$  must be hypercyclic and  $x$  must be a supercyclic vector; however, we now show that the vector  $x$  itself need not be a hypercyclic vector. Thus, the vector  $x$  can have a ‘‘lattice-like’’ orbit—where the orbit comes ‘‘close’’ to every point, but is not dense.

In what follows,  $B$  denotes the backward shift on  $\ell^2$ : For  $(x_0, x_1, x_2, \dots) \in \ell^2$ ,  $B(x_0, x_1, x_2, \dots) = (x_1, x_2, \dots)$ .

**Theorem 2.6.** *If  $T = 2B$  on  $\ell^2$  and  $\epsilon > 0$ , then there exists a vector  $x \in \ell^2$ , such that  $\text{Orb}(T, x)$  is  $\epsilon$ -dense in  $\ell^2$ , but not dense in  $\ell^2$ .*

We begin with a Lemma.

**Lemma 2.7.** *If  $y = (y_0, \dots, y_n) \in \mathbb{C}^{n+1}$ , then there is a vector  $z = (z_0, \frac{z_1}{2}, \dots, \frac{z_n}{2^n})$  such that  $\|z - y\| \leq \frac{1}{\sqrt{3}}$  and  $z_k \in \{w \in \mathbb{C} : |w| \geq 1\} \cup \{0\}$ .*

*Proof.* One can easily check that if  $y = (y_0, \dots, y_n) \in \mathbb{C}^{n+1}$ , then there exists a vector  $z$  having the required form such that  $|\frac{z_k}{2^k} - y_k| \leq \frac{1}{2^{k+1}}$ . It then follows that  $\|z - y\|^2 \leq \sum_{k=0}^n |\frac{z_k}{2^k} - y_k|^2 \leq \sum_{k=0}^n \frac{1}{4^{k+1}} \leq \sum_{k=0}^{\infty} \frac{1}{4^{k+1}} = \frac{1}{3}$ .  $\square$

*Proof of Theorem 2.6.* By Lemma 2.3 it suffices to construct an  $x$  whose orbit comes within a distance  $d$  of every point, for some  $d > 0$ . Let  $F = \{z \in \mathbb{C} : |z| \geq 1\} \cup \{0\}$  and let  $C$  be any countable dense subset of  $F$ . Let  $\{v_n : n \geq 0\}$  be an enumeration of all vectors of finite length with entries in  $C$ . Denote the coordinates of  $v_n$  as  $v_n = (v_n(0), v_n(1), \dots, v_n(k_n))$ . The vector  $x$  will have the following form:

$$x = (v_0, 0, \dots, 0, \frac{v_1(0)}{2^{n_1}}, \frac{v_1(1)}{2^{n_1+1}}, \dots, \frac{v_1(k_1)}{2^{n_1+k_1}}, 0, \dots, 0, \frac{v_2(0)}{2^{n_2}}, \frac{v_2(1)}{2^{n_2+1}}, \dots, \frac{v_2(k_2)}{2^{n_2+k_2}}, \dots)$$

The numbers  $n_k$  are chosen so that  $T^{n_k} x = (v_k(0), \frac{v_k(1)}{2}, \dots)$ . That is,  $n_k$  is precisely the number of coordinates preceding the term  $\frac{v_k(0)}{2^{n_k}}$ . However, before we choose the  $n_k$ 's we first choose  $p_k$ , the number of zeros separating the vectors  $v_{k-1}$  and  $v_k$ . Choose  $p_k$  large enough to satisfy  $\|v_k\|/2^{p_k} < (1/2)^k$ . Now that we know the  $p_k$ 's and the length of the  $v_k$ 's one can determine the  $n_k$ 's.

Notice that

$$\begin{aligned} \|T^{n_j} x - (v_j(0), \frac{v_j(1)}{2}, \frac{v_j(2)}{2^2}, \dots, \frac{v_j(k_j)}{2^{k_j}}, 0, 0, \dots)\|^2 &\leq \sum_{k=j+1}^{\infty} \frac{\|v_k\|^2}{4^{n_k - n_j}} \leq \\ &\leq \sum_{k=j+1}^{\infty} \frac{\|v_k\|^2}{4^{p_k}} \leq \sum_{k=j+1}^{\infty} \frac{1}{4^k} = \frac{1}{(3)(4^j)} < \frac{1}{4^j}. \end{aligned}$$

Thus,

$$\|T^{n_j} x - (v_j(0), \frac{v_j(1)}{2}, \frac{v_j(2)}{2^2}, \dots, \frac{v_j(k_j)}{2^{k_j}}, 0, 0, \dots)\| < \frac{1}{2^j}.$$

For each  $j \geq 0$ , let  $\hat{v}_j = (v_j(0), \frac{v_j(1)}{2}, \frac{v_j(2)}{2^2}, \dots, \frac{v_j(k_j)}{2^{k_j}}, 0, 0, \dots)$ . Thus, we have that  $\|T^{n_j}x - \hat{v}_j\| \leq 1/2^j$ .

Now, suppose  $y \in \ell^2$  and  $y = (y_0, y_1, \dots)$ . Choose an integer  $n \geq 1$  such that  $\|y - (y_0, \dots, y_n, 0, 0, \dots)\| \leq 1$ , then by Lemma 2.7 there exists a vector  $z = (z_0, \frac{z_1}{2}, \dots, \frac{z_n}{2^n}, 0, 0, \dots)$  such that  $\|z - (y_0, \dots, y_n, 0, 0, \dots)\| \leq 1/\sqrt{3}$  and  $z_k \in F$ . Thus, there is a vector  $\hat{v}_j$  that is arbitrarily close to  $z$ , say we choose a  $j$  such that  $\|\hat{v}_j - z\| \leq 1$ . Then we have

$$\|T^{n_j}x - y\| \leq \|T^{n_j}x - \hat{v}_j\| + \|\hat{v}_j - z\| + \|z - y\| \leq \frac{1}{2^j} + 1 + (1 + \frac{1}{\sqrt{3}}) \leq 4.$$

Thus, the orbit of  $x$  comes within a distance 4 of every point in  $\ell^2$ . However, notice that by construction, the first coordinate of  $T^n x$  always belongs to the set  $F = \{z \in \mathbb{C} : |z| \geq 1\} \cup \{0\}$ . In particular, if  $y = (1/2, 0, 0, 0, \dots)$ , then  $\|T^n x - y\| \geq 1/2$  for all  $n \geq 0$ . Hence the orbit of  $x$  is not dense in  $\ell^2$ .  $\square$

**Corollary 2.8.** *If  $T = 2B$  on  $\ell^2$ , then there exist vectors  $x$  and  $y$  such that  $x$  has a dense orbit, the closure of the orbit of  $y$  is compact, and yet  $x + y$  does not have a dense orbit.*

*Proof.* Let  $x$  be the vector constructed as in Theorem 2.6 only with the set  $F = \mathbb{C}$ . Then it is easy to see that  $x$  has a dense orbit. Now, define a vector  $y$  as follows. If  $x = (x_0, x_1, \dots)$  and if  $|x_k| < 1/2^k$ , then let the  $k$ th coordinate of  $y$  be  $(-x_k)$ , otherwise let the  $k$ th coordinate of  $y$  be zero. Then  $x + y$  is a vector of the type constructed in the proof of Theorem 2.6 and hence does not have dense orbit. However, the orbit of  $y$  is contained in the set  $K = \{\{a_k\}_{k=0}^\infty \in \ell^2 : |a_k| \leq \frac{1}{2^k} \text{ for all } k\}$  and it's easy to see that  $K$  is compact.  $\square$

It follows from the Corollary above that there exist two vectors  $x$  and  $y$  such that  $x$  has a precompact orbit,  $y$  has an orbit that is not dense (hence it must be nowhere dense [2]) and yet  $x + y$  has a dense orbit!

**Corollary 2.9.** *If  $T = 2B$  on  $\ell^2$  and  $\epsilon > 0$ , then there exist two vectors  $x, y \in \ell^2$  such that  $\text{Orb}(T, x)$  is dense,  $\|T^n x - T^n y\| \leq \epsilon$  for all  $n \geq 0$ , and yet  $\text{Orb}(T, y)$  is not dense.*

*Proof.* It follows from Corollary 2.8 that there exists two vectors  $z, w \in \ell^2$  such that  $\text{Orb}(T, z)$  is dense,  $\text{Orb}(T, w)$  is compact, and  $\text{Orb}(T, z + w)$  is not dense. Let  $c > 0$  be such that  $\|T^n(cw)\| \leq \epsilon$  for all  $n \geq 0$ . Then choose  $x = cz$  and  $y = (cz + cw)$ .  $\square$

We now show that sufficiently small perturbations of hypercyclic vectors remain hypercyclic.

**Theorem 2.10.** *If  $T \in \mathcal{B}(X)$ ,  $\text{Orb}(T, x)$  is dense in  $X$ , and  $\text{clOrb}(T, y)$  is countable and compact, then  $\text{Orb}(T, x + y)$  is dense in  $X$ .*

*Proof.* Suppose that  $\text{Orb}(T, x)$  is dense in  $X$ , and that  $\text{clOrb}(T, y) = \{v_1, v_2, \dots\}$ . For each  $z \in X$ , there exists a sequence  $n_k \rightarrow \infty$  such that  $T^{n_k}x \rightarrow z$ . Since, the orbit of  $y$  has compact closure, the sequence  $\{T^{n_k}y\}$  has a cluster point. Hence there exists  $m_k \rightarrow \infty$  and a  $v_i$  such that  $T^{m_k}x \rightarrow z$  and  $T^{m_k}y \rightarrow v_i$ . Let  $F_i = \{z \in X : \exists n_k \rightarrow \infty \text{ with } T^{n_k}x \rightarrow z \text{ and } T^{n_k}y \rightarrow v_i\}$ . Then  $X = \bigcup_{i=1}^\infty F_i$ . It follows by the Baire Category Theorem, that for some  $i$ ,  $F_i$  is somewhere dense; that is,  $\text{int}[\text{cl}F_i] \neq \emptyset$ . Fix such a value of  $i$ . Then for any  $z \in F_i$ , there exists a sequence  $n_k \rightarrow \infty$  such that  $T^{n_k}x \rightarrow z$  and such that  $T^{n_k}y \rightarrow v_i$ . Thus,  $T^{n_k}(x + y) \rightarrow (z + v_i)$ .

It follows that  $F_i + v_i \subseteq \text{clOrb}(T, x + y)$ . However,  $F_i + v_i$  is a somewhere dense set, and thus  $\text{Orb}(T, x + y)$  is somewhere dense. It follows that  $\text{Orb}(T, x + y)$  must be dense (see [2]).  $\square$

### 3. FINAL REMARKS

Our work allows us to give a more general form of the well known hypercyclicity criterion. It is an unsolved problem whether this sufficient condition for hypercyclicity is also necessary. It is known that an operator  $T$  satisfies the hypercyclicity criterion (as stated below) if and only if  $T \oplus T$  is hypercyclic (see [1]). Thus the necessity of the criterion is equivalent to the problem of showing that the inflation of a hypercyclic operator is still hypercyclic.

**Theorem 3.1** (The Hypercyclicity Criterion). *Suppose that  $T \in \mathcal{B}(X)$ . If there exists two dense subsets  $Y$  and  $Z$  in  $X$  and a sequence  $\{n_k\}$  such that:*

1.  $T^{n_k}x \rightarrow 0$  for every  $x \in Y$ , and
2. There exists functions  $B_{n_k} : Z \rightarrow X$  such that for every  $x \in Z$ ,  $B_{n_k}x \rightarrow 0$  and  $T^{n_k}B_{n_k}x \rightarrow x$ ,

*then  $T$  is hypercyclic.*

Our Theorem 2.1 allows one to slightly weaken the statement of the criterion.

**Theorem 3.2.** *Suppose that  $T \in \mathcal{B}(X)$ . If there exists two dense subsets  $Y$  and  $Z$  in  $X$ , a  $d > 0$ , and a sequence  $\{n_k\}$  such that:*

1.  $\liminf_k \|T^{n_k}x\| \leq d$  for all  $x \in Y$ , and
2. There exists functions  $B_{n_k} : Z \rightarrow X$  such that for every  $x \in Z$ ,  $B_{n_k}x \rightarrow 0$  and  $\limsup_k \|T^{n_k}B_{n_k}x - x\| \leq d$ ,

*then  $T$  is hypercyclic.*

Notice that the Hypercyclicity Criterion (essentially) corresponds to  $d = 0$ . One may also switch the  $\liminf$  with the  $\limsup$ , however they cannot both be  $\liminf$ 's. The proof of Theorem 3.2 proceeds exactly as the proof of Theorem 3.1 (see [6]) except that one uses Corollary 2.2. Of course Theorem 3.2 is equivalent to Theorem 3.1 because if  $T$  satisfies Theorem 3.2, then  $T \oplus T$  does also. Hence  $T \oplus T$  is hypercyclic and thus  $T$  satisfies Theorem 3.1 (see [1]). Nevertheless, perhaps the weakened form may be useful.

We close with a natural question raised by the author.

**Question 3.3.** If  $X$  is a separable Banach space,  $T \in \mathcal{B}(X)$ , and there exists an  $x \in X$  such that  $\text{Orb}(T, x)$  is dense in the weak topology on  $X$ , then is  $T$  hypercyclic?

It should be noted that the author has constructed a vector whose orbit under twice the Backward shift is weakly dense but not norm dense. Hence a weakly dense orbit need not be norm dense.

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