SOMEWHERE DENSE ORBITS ARE EVERYWHERE DENSE

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Abstract. Let T be a continuous linear operator on a locally convex topological vector space X. We show that if $x \in X$ has orbit under T that is somewhere dense in X, then the orbit of x under T must be everywhere dense in X, answering a question raised by Alfredo Peris.

1. Introduction

A continuous linear operator T on topological vector space X is said to be *hypercyclic* provided there is an $x \in X$ whose orbit under T, $\{x, Tx, T^2x, \dots\}$, is dense in X. We denote the orbit of x under T by $\mathrm{Orb}(T, x)$, and we call $x \in X$ a *hypercyclic vector* for T if $\mathrm{Orb}(T, x)$ is dense in X. Examples of hypercyclic operators include certain adjoint multiplication operators on functional Hilbert spaces (see, e.g., [6]), differentiation on the space of entire holomorphic functions [11], and certain composition operators [4]. For further examples and for an overview of the theory of hypercyclicity, the reader may consult the recent survey article [7] by K.G. Grosse-Erdmann.

The context for our work is provided by the following discussion of two remarkable theorems—Ansari's Theorem and the Multihypercyclicity Theorem. For the remainder of this paper, X will denote an infinite-dimensional topological vector space (over the complex field) that is locally convex and Hausdorff, and T will denote a continuous linear operator on X. Employing a strikingly original argument based on connectivity considerations, Shamim Ansari [1] answered a longstanding question (see [10, Remark 2.13]), proving

> *if T is hypercyclic, then for every positive integer n, the operator T^n is also hypercyclic; moreover, T and T^n share the same collection of hypercyclic vectors.*

A few years before Ansari's Theorem appeared, Domingo Herrero ([8]) raised the following closely related conjecture:

> *if T is multihypercyclic, then T is hypercyclic*

An operator $T : X \to X$ is *multihypercyclic* provided there is a finite subset $\{x_1, x_2, \dots, x_n\}$ of X such that $\cup_{k=1}^n \mathrm{Orb}(T, x_k)$ is dense in X. Recently, George Costakis [5] and Alfredo Peris [13] independently established Herrero's Conjecture, proving that the density in X of $\cup_{k=1}^n \mathrm{Orb}(T, x_k)$ implies

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the density in X of $\text{Orb}(T, x_k)$ for some $k \in \{1, 2, \ldots, n\}$. Peris's proof of this "Multihypercyclicity Theorem" directly exploits an observation made by V. Miller [12], who pointed out that if $\cup_{k=1}^{n} \text{Orb}(T, x_k)$ is dense in X, then for at least one index k with $1 \leq k \leq n$, $\text{Orb}(T, x_k)$ is somewhere dense in X (meaning its closure has nonempty interior). Answering a question raised by Peris ([13, Problem 2]), we prove that if an orbit is somewhere dense, then it is everywhere dense, which provides a new sufficient condition for hypercyclicity and a new proof of Herrero's Conjecture. We emphasize that our "somewhere dense implies everywhere dense" result is new even in the setting of Hilbert space. Also, the reader should note that Ansari's Theorem is a corollary of the fact that multihypercyclic operators are hypercyclic (see, e.g., [7, Section 2c], or the proof of Corollary 2.6 of this paper).

In the next section of this paper, we present our proof that somewhere dense orbits are everywhere dense. In Section 3, we answer another question raised by Peris in [13], this one relating to the notion of supercyclicity. The operator $T : X \to X$ is *supercyclic* provided that there is a one-dimensional subspace of X whose orbit under T is dense in X. We prove that if a one-dimensional subspace has orbit somewhere dense in X, then that orbit is everywhere dense in X. The final section of this paper contains a brief description of connections between our work and the invariant-subset problem for operators on Hilbert space.

2. Somewhere Density and Hypercyclicity

We begin with three lemmas, the first of which is essentially Theorem 4 of [12].

Lemma 2.1. *Suppose that $T : X \to X$ has an orbit that is somewhere dense in X; then for every nonzero polynomial p, the operator $p(T)$ has dense range.*

Proof. Let x have somewhere dense orbit under T, and let F denote the closure of $\text{Orb}(T, x)$. Suppose that for some scalar α, the operator $T - \alpha$ on X fails to have dense range. Then there is a nonzero linear functional Λ that annihilates the range of $T - \alpha$, which yields

$$\Lambda(T^n x) = \alpha^n \Lambda(x)$$

for every positive integer n. Note that Λ cannot annihilate all vectors in the interior of F; hence, the closure of $\{\Lambda(T^n x) : n \geq 1\}$ has interior. However, the closure of $\{\alpha^n \Lambda(x) : n \geq 1\}$ does not. This contradiction shows that for every scalar α, the operator $T - \alpha$ has dense range. Since a product of dense-range operators will have dense range, the proof is complete. □

Recall that $T : X \to X$ is *cyclic* provided it has an orbit with dense linear span, that is, provided that there is an $x \in X$, called a *cyclic vector* for T, such that $\{p(T)x : p \text{ is a polynomial }\}$ is dense in X. The next lemma follows from the observation that any subspace of X containing a nonempty open set must be all of X.

Lemma 2.2. *Suppose that* $\mathrm{Orb}(T, x)$ *is somewhere dense in* X*. Then for each nonnegative integer* j*,* $T^j x$ *is a cyclic vector for* T*.*

Our final lemma is the key to our proof that somewhere dense orbits are everywhere dense.

Lemma 2.3. *Suppose that* $x \in X$ *and that* U *is the interior of the closure of* $\mathrm{Orb}(T, x)$*. Then* $X \setminus U$ *is invariant under* T*.*

Proof. Let F denote the closure of $\mathrm{Orb}(T, x)$ and U the interior of F. Observe that F is invariant under T.

If U is empty, there is nothing to prove. Suppose that U is nonempty. Choose a positive integer j such that $T^j x$ belongs to U and set $x_j = T^j x$. Because $\{T^n x : n \geq j\}$ is dense in the open set U, x_j is a limit point of $\mathrm{Orb}(T, x_j)$ and U is the interior of the closure of $\mathrm{Orb}(T, x_j)$. By Lemma 2.2, x_j is a cyclic vector for T; that is, $\{p(T)x_j : p \text{ is a polynomial}\}$ is dense in X.

Suppose, in order to obtain a contradiction, that T maps some point y in the complement of U into U. Without loss of generality, we may assume y belongs to the complement of F. Because, $X \setminus F$ is open and $\{p(T)x_j : p \text{ is a polynomial}\}$ is dense in X, we may find a polynomial p so that $p(T)x_j$ is close enough to y to ensure (1) $p(T)x_j \in X \setminus F$ and (2) $T(p(T)x_j) \in U$. Because U is contained in the closed T-invariant set F, it follows that the closure of the orbit under T of $T(p(T)x_j)$ belongs to F. However, $\mathrm{Orb}(T, Tp(T)x_j) = \{p(T)T^{j+n}x : n \geq 1\}$. Since x_j is a limit point of $\{T^{j+n}x : n \geq 1\}$, the continuity of $p(T)$ yields $p(T)x_j \in F$. Thus $p(T)x_j$ lies in F and its complement, a contradiction. □

Observe that the preceding lemma shows that if $\mathrm{Orb}(T, x)$ is somewhere dense, then every element of $\mathrm{Orb}(T, x)$, including x itself, belongs to the interior of the closure of $\mathrm{Orb}(T, x)$.

Theorem 2.4. *Suppose that* $x \in X$ *is such that* $\mathrm{Orb}(T, x)$ *is somewhere dense in* X*; then* $\mathrm{Orb}(T, x)$ *is dense in* X*.*

Proof. Let F denote the closure of $\mathrm{Orb}(T, x)$ and let U denote the interior of F, which we are assuming is nonempty. We also assume that F is a proper subset of X for otherwise there is nothing to prove.

The first step entails showing that for every nonzero polynomial p, $p(T)x \notin \partial U$. Suppose, in order to obtain a contradiction, that there exists a nonzero polynomial p such that $p(T)x$ belongs to the boundary of U.

Because U is the interior of the closed set F, $X \setminus U$ is the closure of $X \setminus F$. Thus, because x is a cyclic vector for T, we may choose a collection $\mathcal{Q}$ of polynomials such that $\{q(T)x : q \in \mathcal{Q}\}$ is a dense subset of $X \setminus U$. By Lemma 2.1, $p(T)$ must map the dense set $D := U \cup \{q(T)x : q \in \mathcal{Q}\}$ to a dense set; however, we will show that $p(T)D \subseteq X \setminus U$.

Apply the T invariance of $X \setminus U$ established in Lemma 2.3 to obtain that for each $q \in \mathcal{Q}$, the set $q(T)F$ belongs to $X \setminus U$. Again, using the

T invariance of the complement of U, since $p(T)x \in \partial U \subseteq X \setminus U$, we see that $p(T)F$ belongs to $X \setminus U$. Now, every point in $p(T)D$ is either in $p(T)U \subset p(T)F \subseteq X \setminus U$ or has the form $p(T)q(T)x = q(T)p(T)x$ for some $q \in \mathcal{Q}$. However, $p(T)F$ is a subset of $X \setminus U$ and $q(T)p(T)x$ belongs to $q(T)F$, which is also a subset of $X \setminus U$. Thus, every point of $p(T)D$ belongs to the complement of U, contradicting Lemma 2.1.

Thus, we have proved that for an arbitrary nonzero polynomial p, $p(T)x$ cannot belong to the boundary of U. Because $\{p(T)x : p \text{ is a nonzero polynomial}\}$ is connected, contains points in U (such as points in $\mathrm{Orb}(T, x)$), and contains no boundary point of U, we see that $\{p(T)x : p \text{ is a nonzero polynomial}\}$ is entirely contained in U. However, U is contained in the proper closed subset F of X, which means $\{p(T)x : p \text{ is a nonzero polynomial}\}$ cannot be dense in X, contradicting the fact that x is a cyclic vector for T. Thus our assumption that F, the closure of $\mathrm{Orb}(T, x)$, is a proper subset of X has led to a contradiction, and we conclude that $\mathrm{Orb}(T, x)$ is dense in X, as desired. □

Because a multihypercyclic operator must have a somewhere dense orbit, Theorem 2.4 yields the Multihypercyclicity Theorem, first proven independently by Costakis [5] and Peris [13].

Corollary 2.5 (Herrero's Conjecture)**.** *If T is multihypercyclic, then T is hypercyclic.*

As we noted in the Introduction, the preceding result implies Ansari's Theorem. We include the simple argument (which also may be found in [13], e.g.).

Corollary 2.6 (Ansari's Theorem)**.** *If T is hypercyclic, then for every positive integer n, the operator T^n is also hypercyclic; moreover, T and T^n share the same collection of hypercyclic vectors.*

Proof. Suppose that $T : X \to X$ is hypercyclic and that n is a positive integer. Then T^n is multihypercyclic: $\cup_{k=0}^{n-1}\mathrm{Orb}(T^n, T^k x)$ will be dense in X if x is a hypercyclic vector for T. To see that T^n has the same hypercyclic vectors as does T, simply note that if x is hypercyclic for T, then there will be some j between 0 and $n-1$ such that the vector $T^j x$ is hypercyclic for T^n. Because T must have dense range the set $T^{n-j}[\mathrm{Orb}(T^n, T^j x)] = \mathrm{Orb}(T^n, T^n x)$ will be dense in X, from which it follows that x is also a hypercyclic vector for T^n. □

3. Somewhere Density and Supercyclicity

In this section, we obtain the following analogue of Theorem 2.4.

Theorem 3.1. *Suppose that for some $x \in X$, the set of scalar multiples of elements in $\mathrm{Orb}(T, x)$ is somewhere dense in X; then the set of scalar multiples of elements in $\mathrm{Orb}(T, x)$ is everywhere dense in X so that T is supercyclic.*

The preceding theorem answers a question posed by Peris in [13] that arose in connection with his work on the Multisupercyclicity Theorem. An operator $T : X \to X$ is *multisupercyclic* provided that there is a finite collection of vectors $\{x_1, x_2, \dots, x_n\}$ contained in X such that the set of scalar multiples of elements in $\cup_{k=1}^n \mathrm{Orb}(T, x_k)$ is dense in X. Thus, multisupercyclicity is equivalent to the existence of a finite collection of one-dimensional subspaces of X the union of the orbits of which is dense in X. In [13], Peris proves the following Multisupercyclicity Theorem (establishing a conjecture that Herrero stated in [8]):

if T is multisupercyclic, then T is supercyclic.

Peris's proof is similar to the one he employed to establish the Multihypercyclicity Theorem; in particular, it builds upon Miller's observation that if T is multisupercyclic, then there is a one-dimensional subspace of X whose orbit under T is somewhere dense in X ([12]).

Our proof of Theorem 3.1, which is quite similar to our proof of its hypercyclic analogue, depends on the following lemmas. We use the notation $\langle x \rangle$ to denote the one-dimensional subspace spanned by x.

Lemma 3.2. *Suppose that x is such that $\mathrm{Orb}(T, \langle x \rangle)$ is somewhere dense in X. Then the adjoint T^* of T may have at most one eigenvalue.*

Proof. Our proof uses some ideas contained in the proof of Theorem 3.2 in [2].

Let U denote the interior of the closure of $\mathrm{Orb}(T, \langle x \rangle)$. Suppose, in order to obtain a contradiction, that T^* has two distinct eigenvalues λ_1 and λ_2 with corresponding eigenvectors Λ_1 and Λ_2 respectively. For each positive integer n,

$$\Lambda_1(T^n x) = \lambda_1^n \Lambda_1(x);$$

thus, if either $\lambda_1 = 0$ or $\Lambda_1(x) = 0$, Λ_1 would annihilate the nonempty open set U, which would imply it annihilates all of X because its kernel is a subspace. Thus λ_1 and $\Lambda_1(x)$ are nonzero. The same argument shows that λ_2 and $\Lambda_2(x)$ are nonzero. The preceding observations show that neither Λ_1 nor Λ_2 can annihilate any nonzero element in $\mathrm{Orb}(T, \langle x \rangle)$.

Choose a nonzero scalar α and a positive integer j such that $\alpha T^j x$ belongs to U and set $u = \alpha T^j x$. Note that neither $\Lambda_1(u)$ nor $\Lambda_2(u)$ is zero by the comments in the paragraph above. Because u is an interior point of the closure of $\mathrm{Orb}(T, \langle x \rangle)$, for each neighborhood G of u and each $M > 0$ there will be a nonzero scalar c and an integer $n > M$ such that $cT^n x \in G$. Thus, since Λ_1 and Λ_2 are continuous, there is a sequence (c_j) of nonzero scalars and a subsequence (n_j) of the sequence of positive integers such that

$$\lim_j \Lambda_1(c_j T^{n_j} x) = \Lambda_1(u) \quad \text{and} \quad \lim_j \Lambda_2(c_j T^{n_j} x) = \Lambda_2(u).$$

Set $\beta = \Lambda_1(x)/\Lambda_2(x)$ and $\gamma = \Lambda_1(u)/\Lambda_2(u)$. The quotient sequence

$$\left(\frac{\Lambda_1(c_j T^{n_j} x)}{\Lambda_2(c_j T^{n_j} x)} \right),$$

which equals $(\beta(\lambda_1/\lambda_2)^{n_j})$, converges to the nonzero number γ. It follows that $|\lambda_1| = |\lambda_2|$.

Because λ_1 and λ_2 are distinct eigenvalues, there must be a vector $v \in X$ that is in the kernel of Λ_1 but not in the kernel of Λ_2. Because u is in the open set U, there is an $\epsilon > 0$ such that whenever $|\alpha| < \epsilon$, then $u+\alpha v$ belongs to U. Let α be an arbitrary scalar with $|\alpha| < \epsilon$. Because $u+\alpha v \in U$, we may construct a sequence (b_j) of scalars and a subsequence (k_j) of the sequence of positive integers such that

$$\lim_j \Lambda_1(b_j T^{k_j} x) = \Lambda_1(u + \alpha v) \quad \text{and} \quad \lim_j \Lambda_2(b_j T^{k_j} x) = \Lambda_2(u + \alpha v).$$

Because Λ_1 annihilates v, the first limit tells us that $(b_j \lambda_1^{k_j})$ converges to $\Lambda_1(u)/\Lambda_1(x)$ while the second tells us that $(b_j \lambda_2^{k_j})$ converges to $\Lambda_2(u)/\Lambda_2(x)+\alpha\Lambda_2(v)/\Lambda_2(x)$. Because λ_1 and λ_2 have the same modulus, it follows that

$$\left|\frac{\Lambda_2(u)}{\Lambda_2(x)} + \frac{\alpha\Lambda_2(v)}{\Lambda_2(x)}\right| = \left|\frac{\Lambda_1(u)}{\Lambda_1(x)}\right|.$$

Since α satisfying $|\alpha| < \epsilon$ is arbitrary, we conclude that every element of the open set $\{\Lambda_2(u)/\Lambda_2(x) + \alpha(\Lambda_2(v)/\Lambda_2(x)) : |\alpha| < \epsilon\}$ has the same modulus, which is absurd. □

An immediate consequence of the preceding lemma is that the adjoint of a supercyclic operator has at most one eigenvalue, a fact first proven by Herrero [9] in a Hilbert-space setting. The proof of the next lemma is an easy exercise.

Lemma 3.3. *Suppose that* $\mathrm{Orb}(T, \langle x\rangle)$ *is somewhere dense in* x. *Then for each nonzero scalar* α *and positive integer* j, $\alpha T^j x$ *is a cyclic vector for* T.

Lemma 3.4. *Suppose that* $x \in X$ *and that* U *is the interior of the closure of* $\mathrm{Orb}(T, \langle x\rangle)$. *Then for every nonzero scalar* λ, $X \setminus U$ *is invariant under* λT. *In addition,* $X \setminus U$ *is invariant under multiplication by any scalar.*

Proof. Let F denote the closure of $\mathrm{Orb}(T, \langle x\rangle)$ and U its interior. Observe that for any scalar α, F is invariant under αT.

If U is empty, there is nothing to prove. Suppose that U is nonempty. Choose a positive integer j and a nonzero scalar β such that $\beta T^j x$ belongs to U and set $x_j = \beta T^j x$. For any positive integer k, $\mathrm{Orb}(T, \langle T^k x_j\rangle)$ is dense in the open set U; thus x_j is a limit point of $\mathrm{Orb}(T, \langle T^k x_j\rangle)$ and U is the interior of the closure of $\mathrm{Orb}(T, \langle T^k x_j\rangle)$. By Lemma 3.3, x_j is a cyclic vector for T; that is, $\{p(T)x_j : p \text{ is a polynomial}\}$ is dense in X.

Fix a scalar λ and suppose, in order to obtain a contradiction, that λT maps some point y in the complement of U into U. Without loss of generality, we may assume y belongs to the complement of F. Because, $X\setminus F$ is open and $\{p(T)x_j : p \text{ is a polynomial}\}$ is dense in X, we may find a polynomial p so that $p(T)x_j$ is close enough to y to ensure (1) $p(T)x_j \in X \setminus F$ and (2) $\lambda T(p(T)x_j) \in U$. Because U is contained in the closed set F which invariant under T and multiplication by scalars, it follows that $\mathrm{Orb}(T, \langle \lambda T p(T)x_j\rangle)$

belongs to F. However, $\mathrm{Orb}(T, \langle \lambda T p(T) x_j \rangle) = \mathrm{Orb}(T, \langle p(T) T x_j \rangle)$. Since x_j is a limit point of $\mathrm{Orb}(T, \langle T x_j \rangle)$, the continuity of $p(T)$ yields $p(T)x_j \in F$. Thus $p(T)x_j$ lies in F and its complement, a contradiction.

That $X \setminus U$ is invariant under multiplication by a scalar is elementary: let α be a nonzero scalar; because $\alpha F \subset F$ and $x \mapsto \alpha x$ is a homeomorphism, both U and $X \setminus U$ are preserved under multiplication by α. Because $X \setminus U$ is closed, $\alpha(X \setminus U) \subset X \setminus U$ even if $\alpha = 0$. □

The preceding lemma shows that if $\mathrm{Orb}(T, \langle x \rangle)$ is somewhere dense in X, then each nonzero element of $\mathrm{Orb}(T, \langle x \rangle)$ belongs to the interior of the closure of $\mathrm{Orb}(T, \langle x \rangle)$.

Proof of Theorem 3.1. Let F denote the closure of $\mathrm{Orb}(T, \langle x \rangle)$ and let U denote the interior of F, which we are assuming is nonempty. We also assume that F is a proper subset of X for otherwise there is nothing to prove.

Let W denote the collection of nonzero polynomials not having the (possible) eigenvalue of T^* as a zero. The first step entails showing that for every $p \in W$, $p(T)x \notin \partial U$. Suppose, in order to obtain a contradiction, that there exists a polynomial $p \in W$ such that $p(T)x$ belongs to the boundary of U.

Because U is the interior of the closed set F, $X \setminus U$ is the closure of $X \setminus F$. Thus, because x is a cyclic vector for T, we may choose a collection $\mathcal{Q}$ of polynomials such that $\{q(T)x : q \in \mathcal{Q}\}$ is a dense subset of $X \setminus U$. Apply the T and multiplication-by-scalars invariance of $X \setminus U$ established by Lemma 3.4 to obtain that for each $q \in \mathcal{Q}$, the set $q(T)F$ belongs to $X \setminus U$.

Since $p \in W$, it's easy to see that $p(T)$ has dense range so that $p(T)$ must map the dense set $D := U \cup \{q(T)x : q \in \mathcal{Q}\}$ to a dense set; however, we will show that $p(T)D \subseteq X \setminus U$.

By using the T and multiplication-by-scalars invariance of the complement of U, we see that $p(T)F$ belongs to $X \setminus U$. Now, every point in $p(T)D$ is either in $p(T)U \subset p(T)F \subseteq X \setminus U$ or has the form $p(T)q(T)x = q(T)p(T)x$ for some $q \in \mathcal{Q}$. However, $p(T)F$ is a subset of $X \setminus U$ and $q(T)p(T)x$ belongs to $q(T)F$, which is also a subset of $X \setminus U$. Thus, every point of $p(T)D$ belongs to the complement of U, which implies that $p(T)D$ cannot be dense in X, a contradiction.

We have proved that for all polynomials $p \in W$, $p(T)x$ cannot belong to the boundary of U. Because $\{p(T)x : p \in W\}$ is connected, contains points in U (such as nonzero points in $\mathrm{Orb}(T, \langle x \rangle)$), and contains no boundary point of U, we see that $\{p(T)x : p \in W\}$ is entirely contained in U. Given a coefficient n-tuple $\mathbf{c}$ for any polynomial, there is a sequence of coefficient n-tuples of polynomials in W converging componentwise to $\mathbf{c}$. It follows that $\{p(T)x : p \in W\}$ is dense in X (recall x is cyclic for T). However, this dense set is contained in the proper closed subset F of X, a contradition. Thus our assumption that F, the closure of $\mathrm{Orb}(T, \langle x \rangle)$, is a proper subset of X has led to a contradiction, and we conclude that $\mathrm{Orb}(T, \langle x \rangle)$ is dense in X, as desired. □

Remarks. (a) For simplicity, we have assumed throughout this paper that X is a topological vector space over the complex field. This assumption is not a crucial one. The results and arguments of Section 2 apply, without modification, to the real-scalar setting. To obtain the results of this section under the assumption that scalars are real, one may work with the complexifications of X and T. The adjustments required are essentially similar to those Peris used in [13] to prove multisupercyclicity implies supercyclicity in the real setting.

(b) Jochen Wengenroth [15] has shown that our hypothesis that X be locally convex is also not crucial; that is, Theorems 2.4 and 3.1 are valid in the non-locally convex setting. We thank the referee for providing us with this reference.

4. The Invariant Subset Problem

The study of hypercyclicity is motivated, in part, by its connection to the invariant-subset problem for operators on Hilbert space:

> *Must a continuous linear operator on a Hilbert space H leave invariant a nontrivial closed subset of H?*

(*Nontrivial* here means neither $\{0\}$ nor H.) The corresponding problem in the Banach space setting was solved by C. Read [14], who constructed an operator T on a Banach space X for which every nonzero $x \in X$ is a hypercyclic vector. If the analogous construction were accomplished in Hilbert space, then both the invariant-subset problem and the celebrated invariant-subspace problem would be resolved. For a thorough discussion of the invariant-subset and invariant-subspace problems, the reader may consult [3], especially Chapter 3. Our work shows that the invariant subset problem is equivalent to the following:

> *Must a continuous linear operator on a Hilbert space H have a nontrivial nowhere dense orbit?*

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