1. (a) Does the Mean Value Theorem apply to the functions in the given intervals? If so, find the \( c \) guaranteed by the theorem.

\[ T(\theta) = \tan \theta \text{ on } [0, \pi] \quad f(x) = x + \frac{1}{x} \text{ on } [1, 2] \]

(b) Show that if \( f \) is the quadratic function \( f(x) = \alpha x^2 + \beta x + \gamma \) (\( \alpha \neq 0 \)), then the number \( c \) of the Mean Value Theorem is always the midpoint of the interval \([a, b]\).

(c) Use the Mean Value Theorem to show that \( s = \frac{1}{t^2} \) decreases on any interval to the right of the origin.

(d) Use the Mean Value Theorem to show that \( |\sin x - \sin y| \leq |x - y| \).

(e) Joe travelled 112 miles in 2 hours and claimed he never exceeded 55 miles per hour. Use the Mean Value Theorem to disprove his claim.

2. For the given function and interval, determine the absolute extrema.

(a) \( f(x) = (x - 1)^3(x + 2)^2 \) on \([-2, 2]\)

(b) \( g(x) = \frac{1}{x^2} \) on \([-2, 0)\)

(c) \( h(\theta) = \sin \theta \) on \( \left[ \frac{\pi}{4}, \frac{4\pi}{3} \right] \)
3. (a) A long sheet of metal, 16” wide, is to be turned up at both sides to make a horizontal gutter. How many inches should be turned up on each side for maximum carrying capacity?

(b) Of all right cylinders with a given surface area $S$, find the one with maximum volume. Note: the ends of the cylinders are closed.

(c) A fence, 8’ high, is parallel to the wall of a building and 1’ away from the building. What is the shortest plank that can go over the fence, from the level ground, to prop the wall?

4. For each of the functions below, use the first and second derivatives to analyze the curve. Sketch a graph of the curve, labelling all intercepts, extrema and inflection points.

(a) $f(x) = \sin x - \sin^2 x$ on $[-\pi, \pi]$  

(b) $g(x) = \frac{x - 2}{x - 3}$  

(c) $h(x) = x^4 - 2x$  

(d) $k(x) = \frac{x^2 - 1}{x}$
5. (a) Find the equations of the tangent lines at the inflection points of the graph of $y = x^4 - 6x^3 + 12x^2 - 3x + 1$.

(b) Suppose $x = -1, 0, 2$ are the critical values of the function $f(x)$, and $f''(x) = \frac{(x - 1)(x + 2)}{(x + 3)}$. Characterize the critical values as maxima or minima of $f(x)$.

(c) The graph of $f'$ is given in the figure below. Where is $f$ increasing? Where is $f$ concave down? What are the $x$-values for the maximum values of $f$?