1. Use the figure to compute the following. Justify your answers.

(a) \( \lim_{x \to -4^+} f(x) = \infty \)

(b) \( \lim_{x \to 4^-} f(x) = \infty \)

\[ \lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = \infty \]

(c) \( f(-4) \) is undefined

-4 is not in the domain of \( f(x) \)

(d) \( \lim_{x \to 0^+} f(x) = 1 \)

(e) \( \lim_{x \to 0^-} f(x) = -1 \)

(f) \( f(0) \) is not defined since there is a hole in the graph at \( x=0 \)

(g) \( \lim_{x \to \infty} f(x) = 3 \)

(h) \( \lim_{x \to 3^-} f(x) = 3 \neq 2 = \lim_{x \to 3^+} f(x) \)

(i) Name all points of discontinuity visible of the graph. Which ones are removable?

\[ x = -4, 0, 5 \]

\( x = 0 \) is removable since \( f(x) \) would be continuous at \( x=0 \) if we defined \( f(0) = 1 \).
2. (a) State the \( \varepsilon - \delta \) definition of the limit of a function.

\[
\lim_{x \to a} f(x) = L \quad \text{if for every } \varepsilon > 0 \text{ there is a } \delta > 0 \text{ such that if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \varepsilon.
\]

(b) Use the definition of the limit to show that \( \lim_{x \to -1} (3x + 1) = -2 \).

\[\text{Solution: Let } \varepsilon > 0 \text{ be given. Take } \delta = \frac{\varepsilon}{3}.\]

So if \( 0 < |x - (-1)| < \delta \), then

\[|3x + 1 - (-2)| = |3x + 3| = 3|x + 1| < 3\delta = 3(\frac{\varepsilon}{3}) = \varepsilon.\]
3. Compute the following limits. Remember to show all steps in your calculations.

\( \lim_{x \to 2^+} \frac{x^3 - 5x + 4}{x^2 + 2x - 8} = \lim_{x \to 2^+} \frac{(x-4)(x+1)}{(x+4)(x-2)} = \frac{(-4)(x^+) \cdot 0}{(x^+) \cdot 2} = -\infty \)

(b) \( \lim_{x \to 1} \frac{1 - \sqrt{x}}{x-1} \cdot \frac{1 + \sqrt{x}}{1 + \sqrt{x}} = \lim_{x \to 1} \frac{1 - x}{(x-1)(1+\sqrt{x})} = \lim_{x \to 1} \frac{-1}{1+\sqrt{x}} \)

\[ = \frac{-1}{1+\sqrt{1}} = -\frac{1}{2} \]
(c) \[ \lim_{x \to -\infty} \frac{\sqrt{x^2 + ax + x} - x}{\sqrt{x^2 + ax} - x} = \lim_{x \to -\infty} \frac{ax}{\sqrt{x^2 + ax} - x} = -1 \]

\[ \frac{\sqrt{x^2} - x}{\sqrt{x^2} - x} = 1 \]

\[ \sin \frac{y}{x^2} = \frac{y}{x^2} \]

\[ \lim_{x \to -\infty} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\sin x}{x \cdot \cos x} = 1 \cdot 1 = 1 \]
4. (a) What does it mean for a function to be continuous at \( x = c \)?

\[
\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = f(c)
\]

(b) Find the values of \( a \) and \( b \) so that the function \( f(x) \) given below is continuous for all \( x \).

\[
f(x) = \begin{cases} 
1 - (x + 2)^2 & \text{if } x < -1 \\
ax + b & \text{if } -1 \leq x < 1 \\
x^2 - 4x + 4 & \text{if } x \geq 1
\end{cases}
\]

\[
\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = 1 - (-1 + 2)^2 = 1 - (1)^2 = 0 \\
\lim_{x \to -1^-} f(x) = a(-1) + b = -a + b \\
\lim_{x \to -1^+} f(x) = a(1) + b = a + b
\]

\[
\begin{align*}
- a + b &= 0 \\
\Rightarrow & \quad a = b
\end{align*}
\]

\[
\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = (1)^2 - 4(1) + 4 = 1
\]

\[
\begin{align*}
\text{Let } a + b &= 1 \\
\Rightarrow & \quad a = b = \frac{1}{2}
\end{align*}
\]
5. Find \( k \) so that \( f(x) = kx^2 - (k + 1)x \) has a root in the interval \((3, 6)\). Use the Intermediate Value Theorem to justify your solution.

\[
\begin{align*}
f(3) &= k(3)^2 - (k+1)(3) = 9k - 3k - 3 = 6k - 3 \\
f(6) &= k(6)^2 - (k+1)(6) = 36k - 6k - 6 = 30k - 6
\end{align*}
\]

Need \( f(3) < 0 \) and \( f(6) > 0 \) (or vice versa).

\[
\begin{align*}
6k-3 < 0 & \quad 30k-6 > 0 \\
6k < 3 & \quad 30k > 6 \\
k < \frac{1}{2} & \quad k > \frac{1}{5}
\end{align*}
\]

Any \( k \) from the interval \( \left( \frac{1}{5}, \frac{1}{2} \right) \) will work.

6. Find all holes and asymptotes for the graph of the function \( f(x) = \frac{x^2 + x - 2}{x^3 + 7x + 10} \).

Use limits to justify your response.

\[
f(x) = \frac{(x-1)(x+2)}{(x+5)(x+2)}
\]

Since \( x+2 \) is a factor in both the numerator and denominator, \( x = -2 \) is a hole.

\[
\lim_{x \to -2} f(x) = \lim_{x \to -2} \frac{x-1}{x+5} = 1, \text{ so } y = 1 \text{ is a hor. esg.}
\]

\[
\lim_{x \to -5^+} f(x) = \lim_{x \to -5^+} \frac{x-1}{x+5} = -\infty, \text{ so } x = -5^- \text{ is a vert. esg.}
\]