(1) Colorado is a rectangular state (ignoring the curvature of the earth). Let \( f(x, y) \) by the number of inches of rainfall during 1999 at the point \( f(x, y) \) in that state. What does \( \iint_{\text{CO}} f(x, y) \, dA \) represent? What does this number divided by the area of Colorado represent?

(2) Show that

\[
\int_{0}^{\infty} \int_{0}^{\infty} \frac{1}{(1 + x^2 + y^2)^2} \, dy \, dx = \frac{\pi}{4}.
\]
(3) (a) Consider the ring $A$ of height $2b$ obtained from a ball of radius $a$ when a hole of radius $c$ (with $c < a$) is bored through the center of the ball. Find the volume of $A$.

(b) The centers of two balls of radius $a$ are $2b$ units apart with $b \leq a$. Find the volume of their intersection in terms of $d = a - b$. 
(4) Find the region enclosed by the rose \( r = 4 \cos 3\theta \).

(5) Find the volume of the solid within the cylinder \( x^2 + y^2 = 9 \) and between the planes \( z = 1 \) and \( x + z = 5 \).
(6) Give the five other integrals that are equivalent to

\[
\int_0^2 \int_0^{y^3} \int_0^{y^2} f(x, y, z) \, dz \, dx \, dy.
\]
(7) (a) Use polar coordinates to evaluate
\[ \int_{-a}^{a} \int_{0}^{\sqrt{a^2-y^2}} x^2 y \, dx \, dy. \]

(b) Use spherical coordinates to evaluate
\[ \int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} \, dz \, dx \, dy. \]

(c) Use cylindrical coordinates to evaluate
\[ \int_{0}^{3} \int_{0}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2} \, dz \, dy \, dx. \]
(8) (a) Use the transformation \( x = u^2, y = v^2, z = w^2 \) to find the volume of the region bounded by the surface \( \sqrt{x} + \sqrt{y} + \sqrt{z} = 1 \) and the coordinate planes.

(b) Find the volume of the ellipsoid \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1 \) by making the change of variables \( x = au, y = vb, z = wc \).
(9) (a) Evaluate \( \int \int_D \frac{1}{(x^2 + y^2)^{n/2}} \, dA \), where \( n \) is an integer and \( D \) is the region bounded by the circles with center at the origin and radii \( r \) and \( R \), with \( 0 < r < R \).

(b) For what values of \( n \) does the integral in part (a) have a limit as \( r \to 0^+ \)?
(c) Find \[ \iiint_E \frac{1}{(x^2 + y^2 + z^2)^{n/2}} \, dV, \] where \( E \) is the region bounded by the spheres with center at the origin and radii \( r \) and \( R \), with \( 0 < r < R \).

(d) For what values of \( n \) does the integral in part (c) have a limit as \( r \to 0^+ \)?

“On my honour, I have neither given nor received any unacknowledged aid on this test.”