

Homework 6

Taxes

Econ 101

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1. **Unit Tax.** Suppose that the market for cigarettes is represented by the following demand and supply functions

$$Q_d(p) = a - bp$$

$$Q_s(p) = dp - c$$

where a , b , c and d are all non-negative numbers with $\frac{a}{b} > \frac{c}{d}$. Suppose the government imposed a tax τ per unit where $0 < \tau < \frac{a}{b} - \frac{c}{d}$.

- (a) What is market clearing price for cigarettes, P_0 , and quantity Q_0 , before the tax? **ANSWER** setting demand equal to supply and solving for price we have ...

$$Q_d(p) = Q_s(p)$$

$$\Rightarrow a - bp = dp - c$$

$$\Rightarrow a + c = p(d + b)$$

$$\Rightarrow p = \frac{a + c}{d + b}$$

Plugging that price back into either the demand or supply curve will yield the equilibrium quantity traded ...

$$\begin{aligned}
 Q_d(p = \frac{a+c}{d+b}) &= a - \frac{b(a+c)}{d+b} \\
 &= \frac{ad-bc}{d+b}
 \end{aligned}$$

- (b) What is the elasticity of demand, ϵ , and the elasticity of supply, σ at the pre-tax equilibrium point? **ANSWER.** By definition $\epsilon = \frac{\Delta Q_d P}{\Delta P Q_d}$. Since the demand curve is linear, $-b$ always represents $\frac{\Delta Q}{\Delta P}$. Therefore $\epsilon = \frac{-bP}{Q}$. Plugging in the equilibrium price and quantity derived in the previous question we have

$$\begin{aligned}
 \epsilon &= \frac{-bP}{Q} \\
 &= \frac{-b \frac{a+c}{d+b}}{\frac{ad-bc}{d+b}} \\
 &= \frac{-b(a+c)}{ad-bc}
 \end{aligned}$$

Similarly

$$\begin{aligned}
 \sigma &= \frac{dP}{Q} \\
 &= \frac{d \frac{a+c}{d+b}}{\frac{ad-bc}{d+b}} \\
 &= \frac{d(a+c)}{ad-bc}
 \end{aligned}$$

- (c) What price (P_D) do consumers pay after the tax is imposed? **ANSWER.** In the new tax equilibrium, we must have

$$\begin{aligned}
 Q_d(P_D) &= Q_s(P_D - \tau) \\
 \Rightarrow a - bP_D &= d(P_D - \tau) - c \\
 \Rightarrow a + c + d\tau &= P_D(d + b) \\
 \Rightarrow P_D &= \frac{a + c + d\tau}{d + b}
 \end{aligned}$$

- (d) What price (P_S) do producers get? **ANSWER** Since, in the tax equilibrium, $P_S = P_D - \tau$ we must have

$$\begin{aligned} p_S &= \frac{a + c + d\tau}{d + b} - \tau \\ &= \frac{a + c - b\tau}{d + b} \end{aligned}$$

- (e) What is the excess burden of this tax? **ANSWER** The new quantity traded in the market in the tax equilibrium is $Q_{tax} = Q_d(p = \frac{a+c+d\tau}{d+b}) = a - \frac{b(a+c+d\tau)}{d+b}$ or $Q_{tax} = \frac{ad-bc-bd\tau}{d+b}$. The difference between this quantity and the old (pre-tax) equilibrium quantity is $\frac{bd\tau}{d+b}$. The DWL or excess burden of the tax is equal to one half of the tax rate time this difference in quantity or $DWL = \frac{bd\tau^2}{2(d+b)}$.

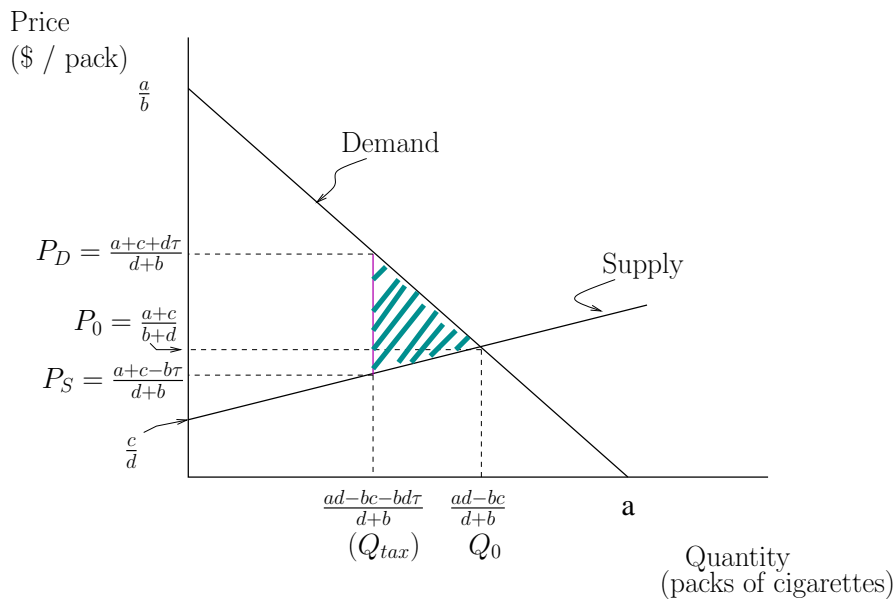


Figure ???. An excise tax. The height of the purple line represent the tax rate, τ . The area of the cyan-shaded area is the DWL or excess burden of the tax. Here we see an example where the slope of the S curve is significantly shallower than the demand curve ($d > |b|$). Hence the burden on consumers is higher than the burden on producers.

- (f) What share of the tax burden do consumers bear? (i.e. what is $\frac{P_d - P_0}{\tau}$ equal to?) **ANSWER.** Given the calculations made above, the consumers' price went up by $\frac{a+c+d\tau}{d+b} - \frac{a+c}{d+b}$ or $\frac{d\tau}{d+b}$. Hence $\frac{P_d - P_0}{\tau} = \frac{d}{d+b}$.
- (g) What would have predicted based on the formula for consumer tax burden share given in class. (i.e. $\frac{P_d - P_0}{\tau} = \frac{\sigma}{\sigma + |\epsilon|}$.) We would have predicted...

$$\begin{aligned}
\frac{P_d - P_0}{\tau} &= \frac{\sigma}{\sigma + |\epsilon|} \\
&= \frac{\frac{d(a+c)}{ad-bc}}{\frac{d(a+c)}{ad-bc} + \left| \frac{-b(a+c)}{ad-bc} \right|} \\
&= \frac{d(a+c)}{d(a+c) + |-b(a+c)|} \\
&= \frac{d}{d+b}
\end{aligned}$$

The same as calculated in the previous part!

(h) How much revenue does the government earn? **ANSWER** $\tau Q_{tax} = \frac{\tau(ad-bc-bd\tau)}{d+b}$.

HINT for all parts: If you are having a difficult time answering these questions in terms of the parameters, a , b , c , and d , try setting them equal to specific values. For example set $a = 20$, $b = 1$, $c = 0$ and $d = 3$. In either case, discuss how changing b and d change the answers to the questions about excess burden and consumers' share of the tax burden. For example what happens to your answers if b goes from 1 to 2, or if d goes from 3 to 4. **ANSWER.** From above the excess burden is equal to $\frac{bd\tau^2}{2(d+b)}$. Notice that if we hold the value of d constant and lower b toward 0, the excess burden goes to 0. Intuitively, this is because lowering the value of b toward zero is akin to making the demand curve go toward perfectly inelastic (vertical). When the demand is perfectly vertical, the quantity traded after the tax does not change and so there can be no DWL. Similarly for supply w.r.t. changes in the value of d .

2. **Monopoly Pricing.** Air Helios has a monopoly on flights from St. Petersburg to Madrid. They face the following marginal willingness to pay curve.

$$MWTP(Q) = 2000 - 10Q$$

where Q is the number of round-trips per week. The firm's cost is

$$C(Q) = 6000 + 200Q + 5Q^2$$

This means that the firm marginal cost is $MC(Q) = 200 + 10Q$.

- (a) If Air Helios is a profit maximizer and must set one price to charge for every ticket it sells, what will that price be. **ANSWER** We need to calculate the marginal revenue function and set it equal to MWTP in order to find the profit maximizing quantity. Then plug that quantity into the MWTP curve to find the profit-max price. For a linear demand curve, marginal revenue has the same vertical intercept and twice the slope. Therefore we have $MR(Q) = 2000 - 20Q$. Setting this equal to MC we get $Q_{\pi-max} = 60$. Plugging that into MWTP we get $P_{\pi-max} = \$1400$.
- (b) How many round-trip tickets will it sell? **ANSWER** 60. See above.
- (c) What is the firm's average cost per ticket at the profit-maximizing ticket amount? **ANSWER.** Average Cost is cost divided by quantity. Therefore $AC(Q) = \frac{6000}{Q} + 200 + 5Q$. Evaluating this at the profit maximizing quantity, we get $AC(Q = 60) = \frac{6000}{60} + 200 + 5 \cdot 60$ or \$600 per ticket.
- (d) How much profit will the firm earn? **ANSWER** $\Pi = (P - AC) * Q = (1400 - 600) * 60$ or \$48K.
- (e) What is the dead-weight loss for this monopoly? **ANSWER** The *efficient* quantity is where $MC = MWTP$, so $Q_{eff} = 90$. The DWL is the difference between the MWTP and MC over the efficient unit not provided (i.e. from $Q=60$ to 90). The value of this loss is therefore $\frac{1}{2}60 * (1400 - 800)$.
- (f) From the cost function we see that this firm's fixed cost amount \$6000 per week. If a different airline had higher fixed costs but faced the same demand curve for its flights, how would that affect its behavior (in terms of price and quantity) and its profit compared with Air Helios. **ANSWER** The profit would of course be lower by the difference in fixed cost, but the behavior - quantity and price - would be exactly the same, at least the short run.

3. **Market Externality** The demand for electricity is given by

$$Q_d(p) = 1000 - 2000P$$

where Q is energy demand in kilowatt-hours per day and prices are quoted in dollars per kilowatt-hour. Suppose that all electricity is produced using coal and that the private marginal

cost of electricity for all coal-fired power plants is \$.05/ kw-h. Meanwhile the marginal cost of environmental damages from coal-fired electricity production is $MEC(Q) = \$.00025Q$.

- (a) What is the *Marginal Social Cost* of electricity? **ANSWER** $MSC(Q) = .05 + .00025Q$.
- (b) What is the efficient level of electricity output? **ANSWER** Note that $MWTP(Q) = \frac{1000-Q}{2000}$. Setting MSC equal to MWTP we have $.05 + .00025Q_{eff} = \frac{1000-Q_{eff}}{2000}$. Multiplying through by 200 we get $100 + .5Q_{eff} = 1000 - Q_{eff}$ or $Q_{eff} = \frac{900}{1.5} = 600$ kw-h.

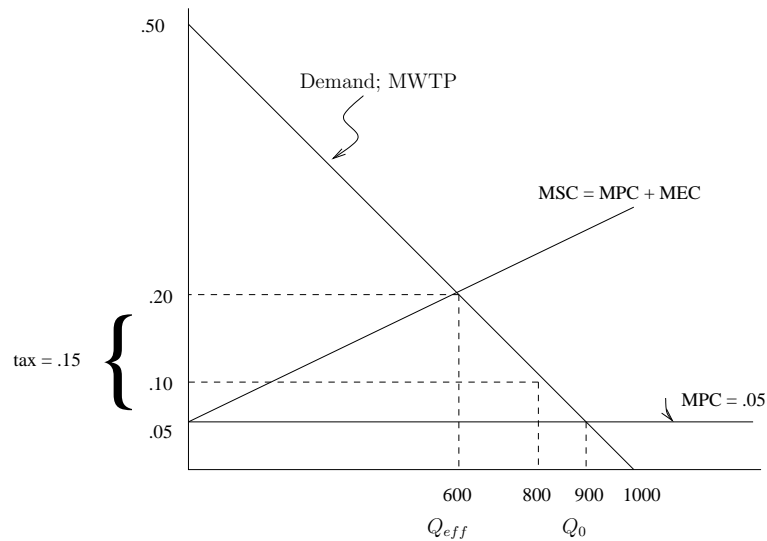


Figure 1.

- (c) Calculate the equilibrium level of electricity output and associate dead-weight loss. **ANSWER** The equilibrium quantity is where private marginal cost equals MWTP. Hence $Q_0 = 900$.
- (d) Propose an electricity tax which achieves the efficient level of output as an equilibrium. **ANSWER** a \$.15 per kw-h excis tax would raise the consumers' price by exactly \$.15 per kw-h to \$.20. (Why do consumer's bear all the burden of such a tax here?) This would decrease demand to exactly the efficient quantity - 600 kwh.
- (e) Suppose that firm can produce electricity using wind turbines for \$.1 per kw-h and that this production process results in negligible environmental damages compared to coal. How would this change your answers? **ANSWER** If the tax was on all electricity, it wouldn't change anything, since coal would still have a cost-advantage over wind. However, if the government levied the tax only on coal-generated electricity and exempted wind-generate electricity, then the price of wind-generate would be \$.10 per kwh while

coal would be \$.20 and all the demand would shift to wind. However this would not be efficient as the first 200 units of coal-generated electricity have lower MSC than wind. Hence the optimal tax on coal-generated would be reduced to \$0.05. This would make wind and coal generated electricity the same price. 200 kwh could come from wind and an additional 600 from wind.¹ Hence demand would only be reduced to 800 (instead of 600).

- (f) Given your revised optimal tax in a world with both coal and wind technologies, what does electricity production look like? How much electricity is produced using coal? How much using wind? **ANSWER** see above.

4. Efficient Pollution Control Two firms, A and B, emit Mercury into the atmosphere as a by-product of their production processes. Firm A currently emits 100 tons while Firm B emits 200 tons. The EPA decides that Mercury emissions are harmful and, to be fair, requires that both firms cut back their emissions by 50%. Cutting back on their mercury emissions will reduce the profitability of both firms according to the following marginal abatement cost schedules.

$$MAC_A(Q_A) = \$80Q_A$$

$$MAC_B(Q_B) = \$100Q_B$$

- (a) Calculate the cost of the pollution reduction for each firm. **ANSWER** abating 50 units costs firm A $\frac{1}{2}(80*50)*50 = \$100,000$. Abating 100 units cost firm B $\frac{1}{2}(100*100)*100 = 500,000$. For a total of \$600,000 in abatement cost for the industry.

¹Actually we can't say how much would come from each source.

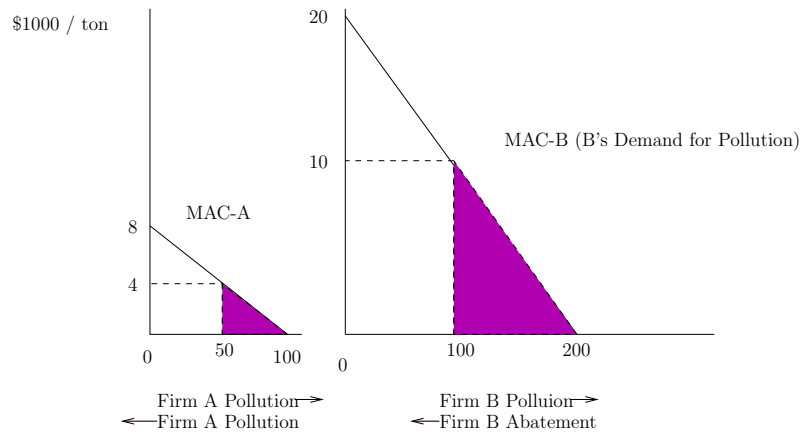


Figure 2. The total abatement cost under the plan in part (a) is the sum of the two purple areas.

- (b) What if, instead, the government granted each firm the right to emit 50% of their historical levels and allowed the firms to trade the permits. Under this plan how much does each firm abate and what does it cost them. What can you say about how much each firm is earning from the sale of permits (or paying out for the purchase of permits)?
- ANSWER.** The firms will trade their permits until their MACs are equal. We know that total abatement in the end must be $Q_A + Q_B = 150$ and $MAC_A(Q_A) = MAC_B(Q_B)$. So combining we have $80(150 - Q_B) = 100 * Q_B$. and we get $Q_B = \frac{12000}{180} = 66.66$ and $Q_A = 83.33$. Putting this all together we conclude that firm A would take its 50 permits and sell 33.33 of them to B. One possible mutually beneficial price would be 6666 per permit. So A would earn roughly \$222,000 revenue in permits sales. (an expenditure for firm B). A's net cost would be $83.33 * 80 * 83.33/2$ in abatement cost minus their permit earnings or about \$56,000 - much less than the \$100,000 they would have had to pay under the 50% mandate in the previous part. Firm B's total cost would be $66.66 * 100 * 66.66/2$ in abatement cost plus the \$222,000 they pay to firm A for the additional permits. or about \$444,000 for a significant saings over their \$500,000 bill in the previous part.

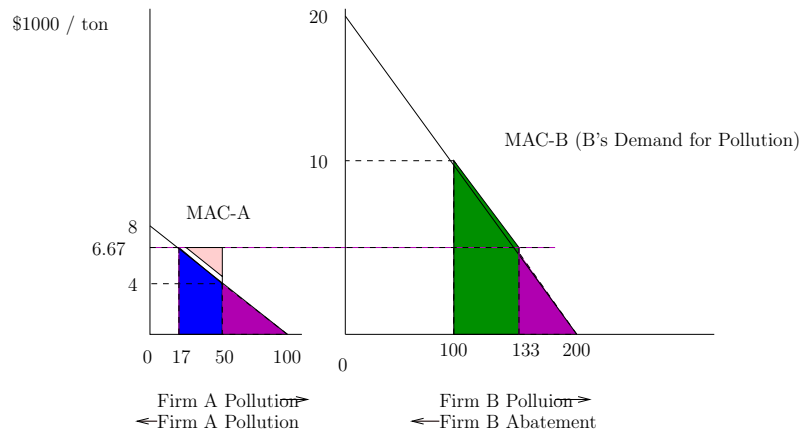


Figure 3. The total abatement cost in part (b) has been reduced to the sum of the now smaller purple area in B's diagram plus the purple and blue areas in A's diagram. The blue area in A's diagram represents the increase in their abatement cost. However they are better off since they gain an amount equal to the blue PLUS the pink area in revenue for selling their permits. The green area represents the reduction in abatement costs from B. They spent the lower rectangular portion of the green areas in permit purchases.

- (c) Propose an emissions charge which achieves the same level of pollution reduction. What is the abatement cost to each firm? How much revenue does the government raise?
ANSWER \$6666 per unit of pollution.