(1) **Unit Tax.** Suppose that the market for cigarettes is represented by the following demand and supply functions

\[ Q_d(p) = a - bp \]
\[ Q_s(p) = dp - c \]

where \( a, b, c \) and \( d \) are all non-negative numbers with \( \frac{a}{b} > \frac{c}{d} \). Suppose the government imposed a tax \( \tau \) per unit where \( 0 < \tau < \frac{a}{b} - \frac{c}{d} \).

(a) What is market clearing price for cigarettes, \( P_0 \), and quantity \( Q_0 \), before the tax?

(b) What is the elasticity of demand, \( \epsilon \), and the elasticity of supply, \( \sigma \) at the pre-tax equilibrium point?

(c) What price \( (P_d) \) do consumers pay after the tax is imposed?

(d) What price \( (P_s) \) do producers get?

(e) What is the excess burden of this tax?

(f) What share of the tax burden do consumers bear? (i.e. what is \( \frac{P_d-P_0}{\tau} \) equal to?)

(g) What would have predicted based on the formula for consumer tax burden share given in class. (i.e. \( \frac{P_d-P_0}{\tau} = \frac{\sigma}{\sigma + |\epsilon|} \)).

(h) How much revenue does the government earn?

**HINT for all parts:** If you are having a difficult time answering these questions in terms of the parameters, \( a, b, c, \) and \( d \), try setting them equal to specific values. For example set \( a = 20, b = 1, c = 0 \) and \( d = 3 \). In either case, discuss how changing \( b \) and \( d \) change the answers to the questions about excess burden and consumers' share of the tax burden. For example what happens to your answers if \( b \) goes from 1 to 2, or if \( d \) goes from 3 to 4.

(2) **Monopoly Pricing.** Air Helios has a monopoly on flights from St. Petersburg to Madrid. They face the following marginal willingness to pay curve.

\[ MWTP(Q) = 2000 - 10Q \]

where \( Q \) is the number of round-trips per week. The firm’s cost is

\[ C(Q) = 6000 + 200Q + 5Q^2 \]
This means that the firm marginal cost is $MC(Q) = 200 + 10Q$.

(a) If Air Helios is a profit maximizer and must set one price to charge for every ticket it sells, what will that price be.

(b) How many round-trip tickets will it sell?

(c) What is the firms average cost per ticket at the profit-maximizing ticket amount?

(d) How much profit will the firm earn?

(e) What is the dead-weight loss for this monopoly?

(f) From the cost function we see that this firm’s fixed cost amount $6000 per week. If a different airline had higher fixed costs but faced the same demand curve for its flights, how would that affect its behavior (in terms of price and quantity) and its profit compared with Air Helios.

(3) Market Externality

The demand for electricity is given by

$$Q_d(p) = 1000 - 2000P$$

where $Q$ is energy demand in kilowatt-hours per day and prices are quoted in dollars per kilowatt-hour. Suppose that all electricity is produced using coal and that the private marginal cost of electricity for all coal-fired power plants is $.05/ kw-h. Meanwhile the marginal cost of environmental damages from coal-fired electricity production is $MEC(Q) = \$0.00025Q$.

(a) What is the Marginal Social Cost of electricity?

(b) What is the efficient level of electricity output?

(c) Calculate the equilibrium level of electricity output and associate dead-weight loss.

(d) Propose an electricity tax which achieves the efficient level of output as an equilibrium.

(e) Suppose that firm can produce electricity using wind turbines for $.1 per kw-h and that this production process results in negligible environmental damages compared to coal. How would this change your answers?

(f) Given your revised optimal tax in a world with both coal and wind technologies, what does electricity production look like? How much electricity is produced using coal? How much using wind?

(4) Efficient Pollution Control

Two firms, A and B, emit Mercury into the atmosphere as a by-product of their production processes. Firm A currently emits 100 tons while Firm B emits 200 tons. The EPA decides that Mercury emissions are harmful and, to be fair, requires that both firms cut back their emissions by 50%. Cutting back on their mercury emissions will reduce the profitability of both firms according to the following marginal abatement cost schedules.

$$MAC_A(Q_A) = \$80Q_A$$
$$MAC_B(Q_B) = \$100Q_B$$

(a) Calculate the cost of the pollution reduction for each firm.
(b) What if, instead, the government granted each firm the right to emit 50% of their historical levels and allowed the firms to trade the permits. Under this plan how much does each firm abate and what does it cost them. What can you say about how much each firm is earning from the sale of permits (or paying out for the purchase of permits)?

(c) Propose an emissions charge which achieves the same level of pollution reduction. What is the abatement cost to each firm? How much revenue does the government raise?