PROBLEM SET 0.
PRELIMINARY MATH EXERCISES
ECON 101

(1) Let $f(x) = 5 - 2x$ and $g(x) = 1 + \frac{x}{4}$

(a) At what value of x does $f(x)$ equal 0? **ANSWER.** Let $x_{f=0}$ be the answer to this question. We can solve for $x_{f=0}$ by setting $f(x)$ equal to zero.

$$\begin{aligned} f(x_{[f=0]}) =& 0 = 5 - 2x_{[f=0]} \\ \Rightarrow& 2x_{[f=0]} = 5 \\ \Rightarrow& x_{[f=0]} = \frac{5}{2} \end{aligned}$$

(b) At what value of x does $g(x)$ equal 11? **ANSWER** Similar to previous. Seting $g(x) = 11$, we get

$$\begin{aligned} g(x) &= 11 \\ \Rightarrow 11 &= 1 + \frac{x}{4} \\ \Rightarrow 44 &= 4 + x \\ \Rightarrow x &= 40 \end{aligned}$$

(c) Draw a diagram depicting the graphs of these functions over the interval of x between 0 and 3. Be sure to label the graphs "f" and "g". Also label all intercepts.

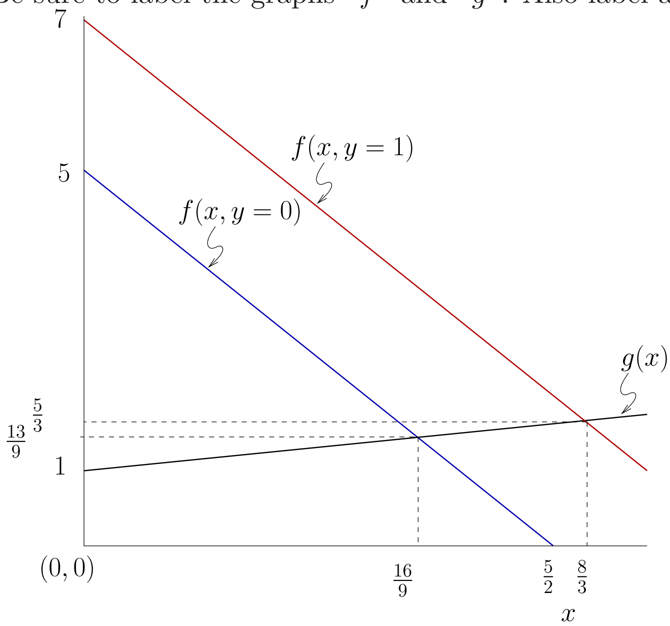

Figure 1. The graphs of $f_{y=0}$ (blue), $f_{y=1}$ (red) and g (black).

(d) What is the slope of the f graph? of the g graph? **ANSWER**. f has a slope of -2 everywhere. It means that from any point along the graph of f, increasing the value of x by 1 *decreases* the value of f by 2. One could alternatively identify the the inverse slope which is everywhere $-\frac{1}{2}$. It means that from any point along the graph of f, increasing the value of f by 1, decreases the value of x by $\frac{1}{2}$. g has a slope of $\frac{1}{4}$ and an inverse slope of 4.

(e) Characterize the intersection of $f(x)$ and $g(x)$ (if there is an intersection). At what value of x do they intersect? What are the values of f and g at the intersection? **ANSWER.** Any x which satisfies $f(x) = g(x)$ will represent the x-componenent of an intersection point.

$$\begin{aligned} &f(x) = g(x) \\ \Rightarrow &5 - 2x = 1 + \frac{x}{4} \\ \Rightarrow &20 - 8x = 4 + x \\ \Rightarrow &9x = 16 \\ \Rightarrow &x = \frac{16}{9} \end{aligned}$$

To find the value of the other component at the intersection, we can plug this solution into either $f(x)$ or $g(x)$ (Why?). This gives us the point $(\frac{16}{9}, \frac{13}{9})$ - the only point in the intersection of the two graphs.

(f) Suppose that f were instead a function of two variables:

$$f(x, y) = 5 - 2x + 2y$$

Add a note to your label of the f function so that it looks like this $f(x|y = 0)$ (or something conveying the same idea). Now draw in the same diagram the graph of $f(x|y = 1)$. How did the intersection of f and g change? **ANSWER** Fixing the value of y at 1, we can write the relationship between x and f as

$$f(x, y = 1) = 7 - 2x$$

The picture of this graph will have the same slope as the previous picture of f but a vertical intercept at 7 instead of 5. (See figure 1 .) Now let's solve for the intersection of $f(x, y = 1)$ and $g(x)$.

$$\begin{aligned} &f(x, y = 1) = g(x) \\ \Rightarrow &7 - 2x = 1 + \frac{x}{4} \\ \Rightarrow &28 - 8x = 4 + x \\ \Rightarrow &9x = 24 \\ \Rightarrow &x = \frac{8}{3} \end{aligned}$$

Plugging $\frac{8}{3}$ back into either formula we get $\frac{5}{3}$. That is, the new intersection point is $(\frac{8}{3}, \frac{5}{3})$.

(2) Problem 15, Parkin Chapter 1, Math Appendix: Demand for baloon rides is presented in the following table as a fucntion of price, and temperature

Price	TEMP	ERAT	URE
($ / ride)	50F	70F	90F
5	32	40	50
10	27	32	40
12	18	27	32
20	10	18	27

(a) Draw a graph to show the relationship between the price and the number of rides. **ANSWER.** Figure 2 show several graphs. Each one shows the relationship between rides and price for a different temperature. Notice that in this picture, price changes are depicted as a *movement along a graph*, while temperature changes are represented as *shifts* in the graph. I've followed the backward economics convention of putting the ostensibly dependent variable on the horizontal axis.

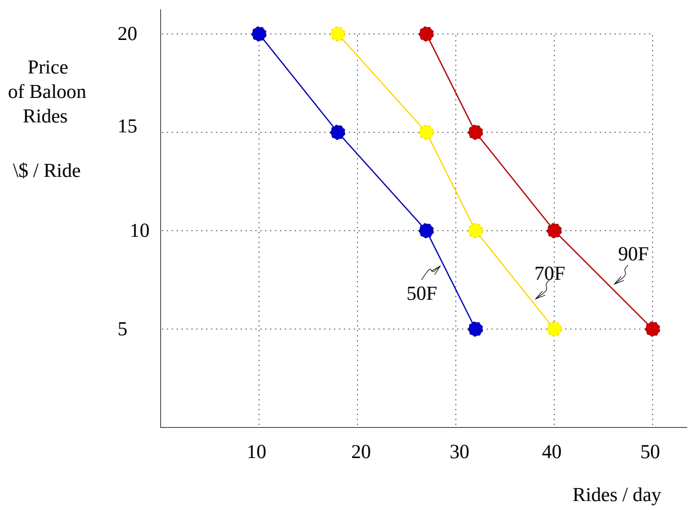

Figure 2. Balloon Rides Versus Price.

(b) Draw a graph to show the relationship between the temperature and the number of rides. **ANSWER.** Figure 3 show several graphs. Each one shows the relationship between rides and temperature for a different price. Notice that in this picture, temperature changes are depicted as a *movement along a graph*, while price changes are represented as *shifts* in the graph. Again, I've followed the backward economics convention of putting the ostensibly dependent variable on the horizontal axis.

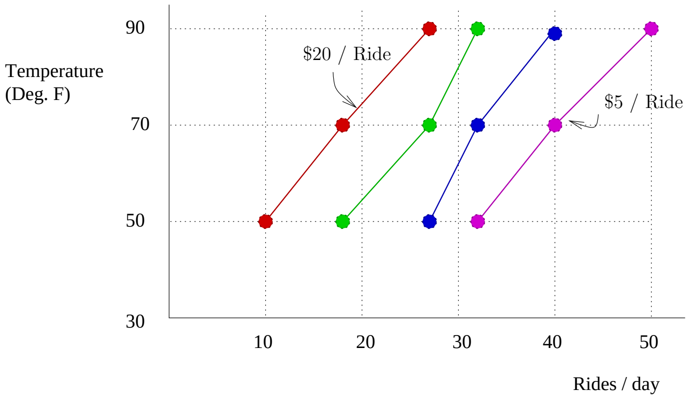

Figure 3. Balloon Rides Versus Temperature.

(c) Draw a graph to show the relationship between the temperature and price. **ANSWER.** Figure 4 show several graphs. Each one shows the relationship between price and temperature for a different level of demand. If we consider balloon rides to be the variable of interest, we can think of these lines (in some cases points) as contour lines. "Uphill" on this contour map is in the direction of lower prices and higher temperatures. Do you think that pattern will hold true for all temperatures? If not, how would extend these contour lines?

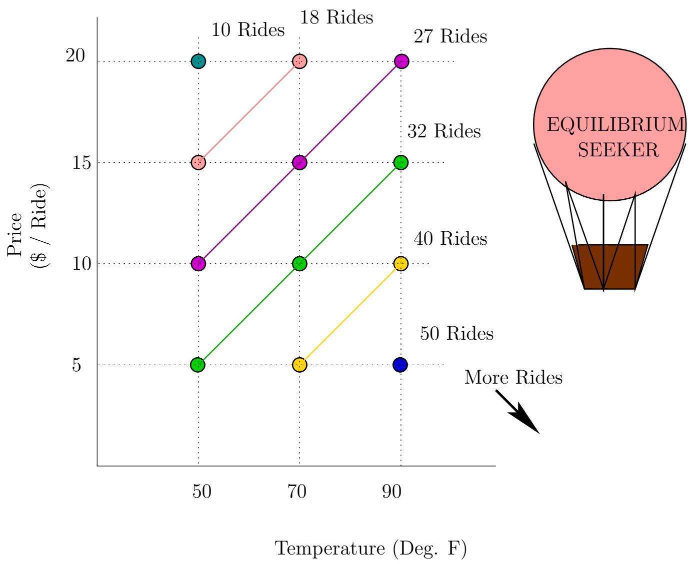

Figure 4. Isodemand Lines in Price-Temperature Space.