MIDTERM EXAM ANSWERS

ECON 210
PROFESSOR GUSE

(1) SHORT ANSWER (30 Points)

(a) (4 points) $x_1(p_1, p_2, m)$ is a consumer's demand for good 1 when the price of good 1 is p_1, the price of good 2 is p_2 and her income is m. Similarly $x_2(p_1, p_2, m)$ is that same consumer's demand for good 2. Assume the consumer lives for only one period and that $p_1x_1(p_1, p_2, m) + p_2x_2(p_1, p_2, m) < m$. Which standard assumption about preferences is violated? Briefly explain. **ANSWER.** *Monotonicity* is violated. A consumer with monotonic (increasing) preferences would always choose a bundle *ON* her budget line. That is, she would spend all of her money. That is not true here. **GRADING NOTE.** 2pts for recognizing that this consumer is not spending all of her income. Another 2pts for connecting that observation to the term "monotonicity".

(b) (3 points) Define *consumption bundle*. **ANSWER** It is an ordered list of quantities where each quantity represents the level of consumption for a particular good or service. In a two-good, one period world with no uncertainty, it is simple a quantity of pizza and a quantity of beer.**GRADING NOTE.** Simply saying "Combination of Goods" is worth 1pt. This answer is imprecise; it misses the idea that a bundle is list of *quantities*. (Apples, Bananas) is a combination of goods. (3 Apples, 5 Bananas) is a consumption bundle. 0pts for any mention of choice, utility, or affordability. A consumption bundle is just a list of quantities. It may or not be affordable, optimal, etc. This distinction was stressed when this was a quiz question. Hence the low tolerance on credit here.

(c) (6 points) Suppose that u represents Emily's preferences for good 1 and good 2 and that $u(x_1, x_2) > u(y_1, y_2)$. Explain briefly why any monotonic transformation of a u would still represent her preferences. **ANSWER**. Define a new utility function v as follows

Date: October 20, 2009.

$$v(z_1, z_2) = f(u(z_1, z_2))$$

If $f : R \to R$ is a strictly increasing function then, by definition of an increasing function, we have $f(u(x_1, x_2)) > f(u(y_1, y_2))$ which means that $v(x_1, x_2) > v(y_1, y_2)$. Hence the entire preference ranking would be preserved under v. (v's ordinal properties are identical to u's).
GRADING NOTES. Many people claimed that a monotonic transformation preserves the ordinal properties *by definition*. That is not true. By definition, a MT is a new utility function defined by wrapping the orginal one in a strictly increasing function. That it preserves the ordinal properties of the original must be shown and was really heart of the question here. Nevertheless I awarded 3 pts to these types of answers by virtue of their recogniztion that it is the ordinal properties which are at stake here.

(d) (3 Points) Assume the decision maker can borrow and lend at rate r. The present value of a \$200 benefit in 4 years followed by a \$300 cost in 7 years is equal to _____.**ANSWER:**

$$NPV = \frac{200}{(1+r)^4} - \frac{300}{(1+r)^7}$$

(e) (3 Points) What is a *luxury good*? **ANSWER:** We say a good is a *luxury* if it is normal (income and demand change in the same direction) and the rate of change in demand for a change income is faster than the change income. In other words, a good is a luxury for an indivual if the income elasticity is greater than 1.

(f) (5 Points) Write down a utility function for some one who consumes cups of coffee and other things. Make it so that when the price of coffee is \$1 their demand for coffee is independent of income.**ANSWER.** One possible answer ...

$$u(z, x_C) = z + \log(x_C)$$

where z is the consumption of all other goods and x_C is the consumption of coffee. Note that when the price of coffee is \$1, demand in this case would be exactly 1 cup of coffee no matter the income level - unless income was actually less than \$1. (in which case demand would be the fraction of a cup which the consumer can purchase with all of his very low income.) **GRADING NOTE.** I was pretty liberal on this question. Some people wrote down perfect substitute preferences. Though PS is

a form of quasilinear, its a pretty bad example, since demand WILL often depend on income. Nevertheless I accepted, since demand does not depend on income for some price combinations (when the consumer isn't consuming any coffee.) I also accepted $u(z, x_C) = z + f(x_C)$. This is also a marginal answer since I have no idea what you mean by f. This equation describes the entire class of quasilinear utility functions, but it is not, by itelf, a utility function. Also I awarded 3 pts partial credit for people who drew a nice picture of quasilinear preferences (hopefully with an income expansion path) but did not actually write down an example of a function which would generate the picture.

(g) (6 Points) George wants to open an iron ore mine. If he does, it will generate \$1M in net revenue for 30 years after which it will be exhausted. At that point it will cost \$500K per year forever to pay someone to prevent and monitor water pollution from the site. Write an expression for the net present value (NPV) of this project. As the interest rate decreases, is this project more or less likely to have a positive NPV? Explain briefly. **ANSWER**. Using the formula for a finite stream of payments, we can write the present value as roughly[1]

$$NPV = \frac{1M}{r}\left(1 - \frac{1}{(1+r)^{30}}\right) - \frac{500K}{r(1+r)^{30}}$$

Note the first set of terms represents the PV of \$1M annually for 30 years. The second set of terms (after the minus sign) represents the present value of \$500K in costs forever, but where the first payment of that cost does not come due for 30 years. This is a project with benefits closer in time than its costs. Hence it will be more attractive the *higher* the interest rate. **EXTENDED ANSWER:** One can combine terms to get

$$NPV = \frac{1M}{r} - \frac{1.5M}{r(1+r)^{30}}$$

Hence the NPV is positive as long as

$$1M > \frac{1.5M}{(1+r)^{30}}$$
$$\Rightarrow r > 1.5^{\frac{1}{30}} - 1$$

[1]

GRADING NOTE: Up to 3 pts for correct NPV formulation and up to 3 pts for explaining why NPV is less likely to be positive. A common error in the NPV formulation was treating the cost stream as though it began next year (instead of in thirty years) writing $\frac{500K}{r}$ for the PV of costs instead of $\frac{500K}{r(1+r)^{30}}$. This was perhaps the most difficult aspect of the problem and I only deducted 1 pt for this error. However, multiple errors in the formulation or errors which revealed a poor understanding of the basic discounting formulas from the homework resulted in 0 out of 3pts for this part of problem. Partial credit for the explanation was rare. Decent answers needed to demonstrate an understanding that the PV of *both* the benefit stream and the cost stream would increase as r decreases. Hence it is the *relative* magnitude of these changes which determines the direction of change in the NPV.

(2) (15 Points) Sally has smooth convex rational preferences for consumption and leisure. Assume that both leisure and consumption are normal goods. Sally has a very flexible employer who lets her work as many hours per week as she likes up to 60 and pays her \$5 per hour. Under the status quo (SQ), Sally is guaranteed government assistance of \$100 per week.
   (a) (5 points) Draw Sally's budget under the status quo.
   (b) (1 points) Under the status quo, Sally chooses to work 20 hours per week. Mark this choice in your diagram and label it "SQ".
   (c) (5 points) Tad Chiggrin complains that the government should not have to pay for welfare benefits for people like Sally who earn their own income. He proposes welfare reform plan "C" where a welfare recipient's assistance is reduced by \$1 for each dollar that person earns and predicts that his proposal will remove people like Sally from the welfare rolls saving the government millions of dollars. Draw Sally's budget under plan "C" in the same diagram.
   (d) (4 points) A prominent liberal lawmaker Saul Goodrock says the Mr. Chiggrin's proposal will result in poverty for people like Sally and moreover plan 'C' is unlikely to save the government any money. Is Goodrock's critique valid? Explain. **ANSWER** Senator Goodrock could be exactly correct *if* Sally's preferences for consumption and leisure are similar to the those represented by the red indifference curve in the diagram. Since Sally's effective wage under plan 'C' is zero for the first 20 hours of work. It seems reasonable that she might lose all incentive to work. (See point 'C-2' in Figure 1.) In this case, her consumption would drop in half to \$100 and the government would save exactly nothing on her welfare

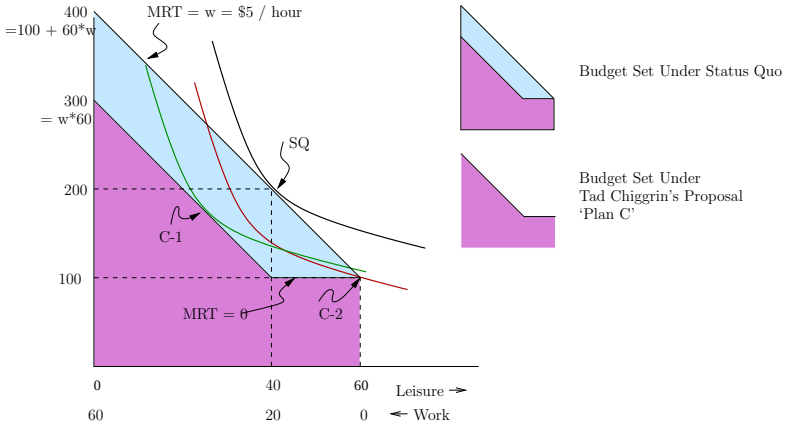

FIGURE 1. The only thing we know for sure about Sally's preferences is that 20 hours of work is an optimal choice under the status quo and that both goods - consumption and leisure - are normal. The black indifference curve along with the green one are meant to represent one possibility for Sally's preferences consistent with these assumptions. This "green" possibility suggests that Sally will increase the number of hours she works under Plan 'C'. The black indifference curve along with the red one are meant to represent another qualitatively different possibility. This "red" possibility suggests that Sally might actually decide to not work at all under Plan C. Both are plausible given the facts in the problem.

checks. On the other hand, the truth could be closer to what Representative Chiggrin had in mind *if* Sally's preferences are more like those represented by the green indifference curve in the diagram. In this case, Sally, chooses to work more hours under plan 'C'. (See point 'C-1' in Figure 1.) That must mean more than 20 hours and so the government's cost in terms of welfare spending would drop to zero. However, because both goods are normal, we know that she would never work more than 40 hours per week under plan 'C' and therefore must be making a total of less than $200 - her original income under the status quo. Perhaps, it is interesting, then, to note that the government will either save nothing on welfare spending on Sally or will save the entire amount of $100. Nothing in between is possible given our assumptions. However, if we consider that the population is likely to be filled with a variety of preferences - some more like the red Sally and some more like the green Sally - then total government spending will go down to something greater than zero and welfare recipients will all be worse off. Possible

Final Exam Question: How does the Earned Income Tax Credit address competing concerns in the welfare debate of providing incentive to work while relieving poverty? Is the EITC the best solution you can think of? **GRADING NOTE:** MOst people got 2 pts out of 4 on this part for at least mentioning Sally's lack of incentive to work any amount on $(0, 20]$. I reserved full credit for explanations which successfully used the normality assumption to draw conclusion about how much she will now work and earn (if at all, see above).

(3) (20 points) Household A has nice rational preferences for rice, x_r, and fish, x_f. The households weekly demand for rice and fish are given by the functions $x_r(p_r, p_f, m)$ and $x_f(p_r, p_f, m)$ repsectively, where p_r is the price of rice, p_f is the price of fish and m stands for the A's weekly income.

(a) (10 points) Suppose that the price of fish is fixed at $\bar{p_f}$ and A's income is $\bar{m}$. When the price of rice decreases from p_r^H to p_r^L, the demand for rice falls from $x_r(p_r^H, \bar{p_f}, \bar{m})$ to $x_r(p_r^L, \bar{p_f}, \bar{m})$. In other words assume that $p_r^L < p_r^H$ and $x_r(p_r^L, \bar{p_f}, \bar{m}) < x_r(p_r^H, \bar{p_f}, \bar{m})$. Consider what happened to A's demand for *fish* when the price of rice decreases from p_r^H to p_r^L as just described. Which of the following is true.

- $x_f(p_r^L, \bar{p_f}, \bar{m}) < x_f(p_r^H, \bar{p_f}, \bar{m})$
- $x_f(p_r^L, \bar{p_f}, \bar{m}) > x_f(p_r^H, \bar{p_f}, \bar{m})$
- $x_f(p_r^L, \bar{p_f}, \bar{m}) = x_f(p_r^H, \bar{p_f}, \bar{m})$

Explain your answer using a diagram if necessary. What can you say about the income and substitution effects on the demand for fish?
ANSWER: $x_f(p_r^L, \bar{p_f}, \bar{m}) > x_f(p_r^H, \bar{p_f}, \bar{m})$. By the observation given in the problem, rice is a Giffen good. The demand *decreased* when its price decreased. However a price decrease results in a expansion of the budget set. This means that whatever bundle the household was choosing when the price of rice was high, they can *still* afford. The monotonicity assumption tells us that they definitely will not choose a bundle involving less of both goods. Since they are choosing one with less rice, it means the new choice *must* have a greater quantity of fish. Moreover, by the law of compensated demand, the substitution effect on fish for price decrease in rice, must be negative. Since we just established that the total effect must be positive, it means that the income effect on fish must be positive and its magnitude must be greater than the SE. **GRADING NOTE:** 5 pts for saying that for successfully arguing that demand for

fish must increase. Another 5 pts for the decomposition of the change in fish demand into SE and IE.

(b) (10 points) Suppose that in addition to the decrease in the price of rice, income changed to m' defined as follows.

$$m' = p_r^L x_r(p_r^H, \bar{p_f}, \bar{m}) + \bar{p_f} x_f(p_r^H, \bar{p_f}, \bar{m})$$

Does A prefer the budget with paramters $(p_r, p_f, m) = (p_r^H, \bar{p_f}, \bar{m})$, or would they rather face the budget given by parameters $(p_r, p_f, m) = (p_r^L, \bar{p_f}, m')$. Explain your answer using a diagram, if need be.
ANSWER: A weakly prefers $(p_r^L, \bar{p_f}, m')$ and probably strongly prefers it...m' is exactly the amount income needed to buy the original choice at the new prices (the set of prices with a low price of rice, p^L). In other words, $(p_r^L, \bar{p_f}, m')$ is our familiar "compensated budget", while $(p_r^H, \bar{p_f}, \bar{m})$ is just our original budget. By design the Slutsky-compensated budget cannot make the household worse off, and probably would make them strictly better off. **GRADING NOTE:** 3 pts for recognizing that $(p_r^L, \bar{p_f}, m')$ was the compensated budget line. Another 7 pts for explaining why comp'd budget reaches at least as high IC.

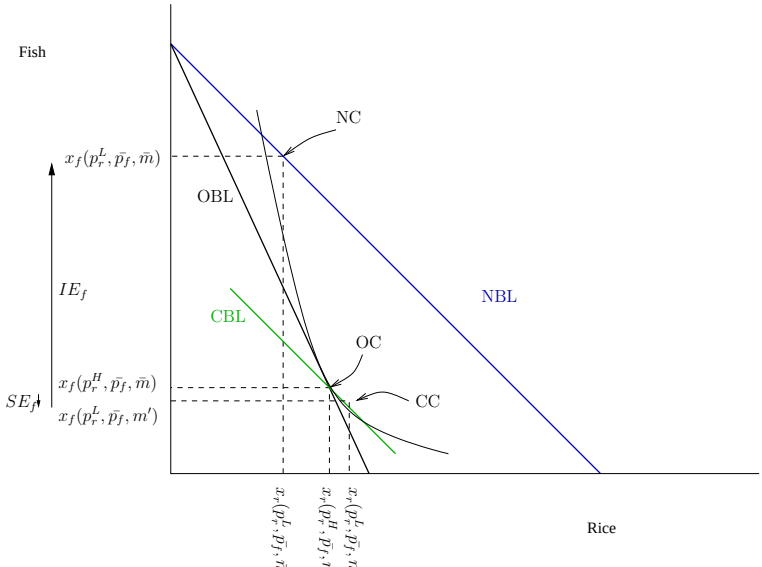

FIGURE 2. OC = Original Choice; NC = New Choice; CC = Possible Compensated Choice (Consistent with LOCD).

(4) (10 Points) *Interest Rate Hike.* Consider an agent who cares about current consumption and future consumption. Assume that current and future consumption are both normal goods for this agent. The consumer has an

endowment $e = (e^0, e^1)$ where e^0 is his current income and e^1 is his future income. Let $c^0(r, e)$ be this agent's demand for current consumption as a function of the interest rate r and his endowment, e.

(a) (5 Points) When the interest rate is equal to r^L, this agent borrows. In other words, $c^0(r^L, e) > e^0$. Draw this choice in a well-labeled diagram. **ANSWER** see Figure 3. **GRADING NOTE:** "well-labeled" is admittedly vague, but for full credit your diagram should be labeled well enough that it speaks (easily) for itself. Labeled axes, labeled interecepts, labels for the endowment and the choice, annotation on the slope of the budget line, etc - the typically cast of characters for our consumption bundle space diagrams.

(b) (5 Points) Suppose the interest rate is instead $r^H > r^L$. Assuming nothing more than normality, monotonicity and the law of compensated demand, what can you say about the consumer's new choice? In other words did current consumption go up or down? Did future consumption go up or down? Explain. **ANSWER**. One possible case is shown in Figure 3. By the LOCD, The substitution effect of the interest rate interest must be negative on current consumption and positive on future consumption. Also, by the normality assumption, the income effect must be negative along both dimensions. One could discuss effects on

(i) *Current and Future Consumption Levels.* The effect of the rate change on current consumption is clear; since both SE and IE are negative along this dimension, current consumption must decrease. However since the SE and IE work in opposite direction along the future consumption axis, the effect on future consumption is ambiguous. In general, the SE on future consumption is more likely to dominate the IE if the rate change is more extreme or if his initial level of borrowing was relatively small.

(ii) *Likelihood of Remaining a Borrower?* One might also comment on whether the consumer remains a borrower or becomes a lender. This go either way as well. Again, he would be more likely to become a lender if the rate increase is extreme or if the initial level of borrowing was modest. Figure 3 shows a consumer for whome it is optimal to neither borrow nor lend at the higher interest rate. Hence a little bit higher rate would induce this consumer to become a lender and a slighter lower rate (than r^H) would have kept this consumer a borrower.

(iii) *Welfare.* Typically one would expect a rate increase to make a borrower worse off. Certainly that is true, if he remains a borrower. However, if the rate increase is extreme or the initial level

of borrowing was very small, it is conceivable that he not only would become a lender, but would be better off as well. Note this is *not* the case in Figure 3. The consumer's response is to almost becomes a lender, but he is strictly worse off.

GRADING NOTE: An answer which fully explained (in terms of SE and IE) why current consumption must decrease and the effect on future consumption is ambiguous was worth full credit 5pt. Only fully explaining the effect on current consumption was worth 2pts. Partial credit could also be earned for showing a good picture but it had to be explained for full credit. Also in general I marked +1 for saying reasonable well explained or demonstrated things about the consumer's status as a borrower or lender and her welfare. Conversely I marked -1 for saying wrong things.

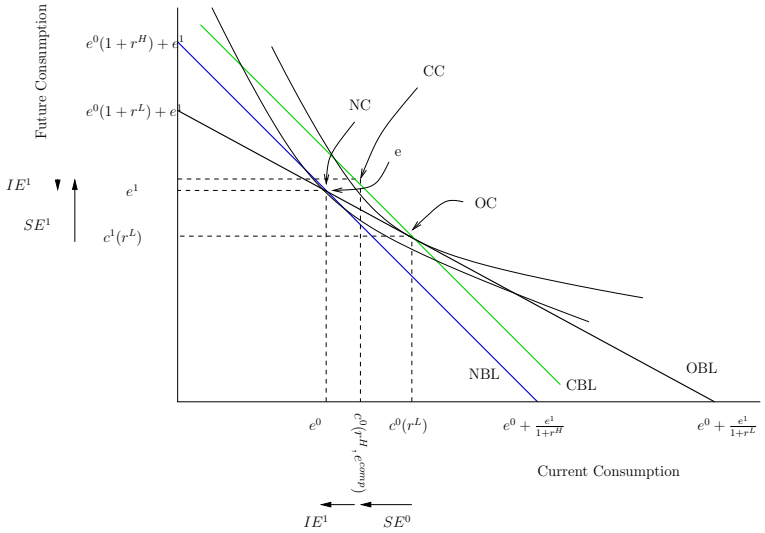

FIGURE 3. One Possible Response by a Borrower to an Interest Rate Increase: Become Polonius.