
Prof. Guse, W & L University

Instructions. You have 3 hours to complete the exam. There are 100 points available. Please write your responses on the exam itself in the space provided. If you require additional space, write on the back of the page. You may refer only to your own handwritten, “cheat sheet”. Calculators and all other references materials are not allowed. If a question asks for a numeric quantity you may leave your answer in expression form for full credit. (e.g. \( \frac{40-30}{5} \) would be perfectly acceptable in place of “2”.) Be sure to label any diagrams you draw, to show your work and to explain your reasoning. You may keep your cheat sheets. Thank you and good luck!

Name:

Pledge:
SHORT ANSWER (30 Points)

1. (4 Points) Define \textit{conditional factor demands} and describe how to derive them from a production function. **ANSWER** The set of Conditional Factor Demands is the solution to the Cost Minimization Problem. That is, CFDs are the inputs that minimize cost conditional on being sufficient to produce a target output level.

2. (2 Points) What is the relationship between conditional factor demands and the cost function? **ANSWER.** Suppose that $L(y, p_L, p_K)$ and $K(y, p_L, p_K)$ are the conditional factor demand functions - mapping to the cost minimizing input bundles for any given target output level $y$. Then the cost function is $c(y, p_L, p_K) = p_L L(y, p_L, p_K) + p_K K(y, p_L, p_K)$. In other words, the cost function is the cost of the conditional factor demands.

3. (4 Points) Given a production function $f(L, K)$, how would you find the profit-maximizing level of output in the short run when $K$ is fixed at $\bar{K}$? **ANSWER** When $K$ is fixed at $\bar{K}$, the best a firm can do is to produce where the marginal revenue product of $L$ is equal to its price: $P_L = P_{\text{output}} \frac{\partial f(L, \bar{K})}{\partial L}$. Put another way, the firm should be at the point where the slope of the production function $\left( \frac{\partial f(L, \bar{K})}{\partial L} \right)$ is equal to the slope of the iso-profit lines $\left( \frac{P_L}{P_{\text{output}}} \right)$.

4. The average cost curve for a utility monopoly lies everywhere above the demand curve for its service.

   (a) (5 Points) What is the break-even price? **ANSWER** The break-even price does not exist.

   (b) (5 Points) Is it possible for this monopoly to make a profit? **ANSWER** see the answer to question number 2 on Homework 8, in particular part (d).
5. (2 points) Name two reasons why long run supply is more elastic than short run supply. **Answer**

- **Intensive Margin:** because individual firms’ long run marginal cost curves are typically flatter than their short run marginal cost curves. This is due (ultimately) to the fact that more inputs may be changed in the LR than in the SR (by definition).
- **Extensive Margin:** LR Average Cost curves will lie above the relevant SR Average Variable cost for firms. Therefore LR break even price will be higher than SR Shutdown prices. Hence for a given price increase (/decrease) more firms will enter (/exit) in the LR than will resume (/shutdown) in the SR.

6. (5 Points) Paul and Carol face the same prices for pizza and beer. Paul drinks 5 beers per week and eats 5 pizzas. Carol drinks 15 beers per week and eats 3 pizzas. Is Paul’s Marginal Rate of Substitution of Pizza for Beer greater than, less than or equal to Carol’s. Explain. **Answer** They must have equal marginal rates of substitution. Since they are both consuming positive amounts of both, neither can be at a corner solution. Therefore, they must both be consuming at a point where \( \text{MRS} = \text{MRT} \). Further, since they both face the same prices for these two goods, the slopes of their respective budget lines must be the same (MRT).

![Budget Lines](image)

Figure 1. Here we see Paul and Carol’s consumer problem diagrams drawn on top on each other. Paul’s indifference curves are drawn in blue, Carol’s in Red.

7. (3 Points) A firm’s technology has constant returns to scale. Describe the shape of its long run marginal cost curve. Explain.
8. (20 Points) **A Partial Price Floor.** If the government imposes an unsupported price floor only a portion of consumers in a market, it could result in a surplus *gain* for some consumers. Explain using a diagram. **ANSWER** An example of this might be a minimum wage which is set on only a part of a labor market. This was exactly one of the question on the final exam from Winter 06 whose answer I reproduce below.

Suppose that the city council in Lexington decides to pass a “living wage” law which specifies that all employees working in the city of must be paid $10 per hour. Meanwhile jobs in the surrounding county are subject only to the federal minimum wage law of $4 per hour. Make the following assumptions where needed.

- Assume that before the passage of the “living wage” law, the market-clearing wage throughout the entire region (city and county) was $7 per hour.
- Assume that this a perfectly homogeneous labor market in the sense that all employees and all the hours they work are perfect substitutes from employers’ point of view.
- Assume that workers are mobile, easily able to shift from jobs in the city to the county and vice versa. Employers and jobs are *not* mobile.

The county demand curve remains the same, the city-demand curve essentially looks perfectly inelastic at wages below $10 from the point of view of the county. Hence the living wage law has the effect of constrasting aggregate demand for labor at wages below floor price of $10. The supply curve of labor remains unaffected by the law. Therefore, wages in the county are depressed to $w_{new} < $7 and employment in the county increases. Wages in the city are, of course by law increased to $10 and employment there decreases.¹

¹Note that it may be possible to construct a situation in which the living wage law pushes so worker out of the city that the federal minimum wage become binding. This is not happening in my diagram.
Since the price (wage) goes down for some (county) consumers (employers), they gain. This answers the question. What follows is an analysis of changes in employee (producer) surplus. The effect of the assumptions here is that no worker is contributing hours to the labor market which they value in leisure at more than $7 per hour - even those who are lucky enough to be getting paid $10. To analyse the change in employee or worker surplus, we have to look at several possible outcomes. Consider the hours being worked in the city. For the workers who are lucky enough to work these hours, there is a gain of $(10 − 7)D_{\text{Lex}}(10)$ indicated by the blue shaded area in Figure 2. However $D_{\text{Lex}}(10) < D_{\text{Lex}}(7)$ which means that fewer hours are being worked in the city as observed in the previous part. Some of those hours move out into the county (which is the source of the downward pressure on wages there). However, overall employment decreases, so some of those hours simply disappear from the labor market. The lost employee surplus from those disappearing hours is indicated by the solid red area. Notice the horizontal length of this area represents the difference between
total employment before and after the LW law. The hours that move from the city out to the county must accept a lower wage as do all the hours that were already working in the county before the law change. The surplus lost on these county hours (both extant and newly arrived) is the red shaded area. Are workers better off? Yes the workers who continue to work in the city are better off. Meanwhile the workers who were working in the county and the worker who were forced to move to the county or go unemployed are worse off. Whether or not the surplus gained by the winning employees exceeds the surplus lost by the losing employees of this policy will depend primarily on the elasticity of city demand for labor. For instance, if city labor demand is highly elastic, city employers will reduced their demand a lot making the winners’ surplus gain smaller and pushing more hours out into the surrounding county which puts great downward pressure on wages there making the losers’ losses big. In this case, the loss is likely to exceed the gain to employees.
9. **Abating CO\textsubscript{2}**. Time magazine recently published “51 Things We can Do About Global Warming”. Number 5 on their list is “Pay A Carbon Tax”.

(a) (20 Points) Here is an excerpt from the article.

Supporters of the tax argue that a cap-and-trade system\footnote{“cap-and-trade” refers to a policy where a quota would be set on the level of CO\textsubscript{2} emissions and tradable permits representing that level would be allocated (somehow) to polluters. Releasing CO\textsubscript{2} without a permit would result in some kind of penalty- (e.g. a fine)}, especially one that would be global enough to mitigate the 8 billion tons of carbon the world now emits, would be too difficult to administer and too easily gamed by industries looking to sidestep emissions caps. Cap-and-trade advocates counter that like all other flat taxes, a carbon levy would disproportionately burden lower-income families, who spend a greater percentage of their income on energy than rich households.

Use diagrams to compare a tax and a cap-and-trade system which result in the same level of carbon emissions. Discuss the difference between these two policies with respect to burden on lower-income families.\textbf{ANSWER} There are several way one could approach this. Here is the simplest way I’ve thought of to explain the difference. Let’s take as given that there is demand for CO\textsubscript{2} - not because consumers benefit from consuming CO\textsubscript{2} per se, but because the production of services which consumers do care about (transportation, heating, electricity, etc) is facilitated by using carbon-based fuels. In other words, we can think of the demand for carbon like the demand for any other input - albeit an input with potential large negative externalities, but that shouldn’t affect the private demand for it.

First consider a tax or unit charge, $\tau$, on the carbon content of fuels which results in the demand for CO\textsubscript{2} dropping from $Q_0$, the unregulated quantity to $Q_{\text{target}}$ the level of carbon emissions which the government would like to see. The demand curve alone determines the necessary level of the tax to achieve the target. In other words the tax must be set at the Marginal Willingness To Pay evaluated at $Q_{\text{target}}$. See Figure 3. No assumptions on the demand for carbon (other perhaps than it is downward sloping) are necessary to make this claim.
Second, consider imposing a quota with tradable permits. Since the permits are tradable we expect them to end up in the hand of those willing to pay the most for them. Therefore the final price for permits in the market must be the Marginal Willingness to Pay evaluated at \( Q_{\text{target}} \) - *exactly the same as the tax*\(^3\)

How will consumers be affected? Since both systems must increase the price of carbon to exactly the same level, consumers of products and services like gasoline and electricity which are produced using carbon-based fuels should be indifferece between the two systems. In other words the post-tax demander price for gasoline will be exactly the same as the post-CNT demand price. Hence the loss in consumer surplus will be indentical under each system. See Figure 2.

\(^3\)We are assuming no difference in general equilibrium effects. In particular the demand curve for carbon posited here has some *ceteris paribus* assumptions behind it. In particular we are holding income constant. Since the quota rents in a c-n-t system could potentially be distributed very differently than the revenue in a tax system, there could conceivably be a different prices for carbon accross the two systems.
Figure 4. **Consumer Welfare Loss Under BOTH Systems.** The loss in consumer welfare in either system will be felt through increased prices in goods and service that use carbon as an input. This can be represented as an upward shift in the supply curves of these goods - due to the increase in the price of an input: CO\(_2\). Since both systems must raise the price of carbon to the same level, the loss felt by consumers should be the same.

Will there be any difference in the experience of poor household under the two systems? The analysis above suggests that the answer may be “No.” since poor households will be experiencing the same final post-tax and post-CNT prices for all carbon-intensive goods. However there are some distribution issues to address. In the case of a carbon tax, government revenue will be generated. In the case of a CNT system, there is the permits - whose market value should be equal to the government revenue from a tax. Theoretically the permits and the revenue could be distributed in exactly the same way rendering the two system identical in distribution effects generally and in how they treat poor households in particular. For example, the permits could be allocated to the U.S. Treasury, who in turn could auction them off and generate at least as much revenue as a tax. In this case, the two systems would be very difficult to distinguish in any meaningful way other than administratively. As another example, suppose that the revenue from a tax were rebated in equal shares to every citizen and that under the CNT, an equal number of CO\(_2\) permits were given to every citizen. Then, again, the two system would be practically indistinguishable. However, as noted in the problem, most CNT proposals call for doling out the permits to historic polluters while no CO\(_2\) tax proposal I’ve ever heard of calls for rebating the revenue back to polluters. On the contrary most CO\(_2\) tax proposals call for using the revenue to reduce some widely paid other tax - such as payroll taxes. Under these assumptions the CO\(_2\) Tax clearly benefits poor household MUCH more than a CNT trade sytem. In fact, a CO\(_2\) tax whose revenue goes to reduce labor taxes should benefit almost *all* household more than CNT systems where the permits are doled out to polluters. The exception would be households who happen to own a lot of stock in carbon-intensive firms (e.g. Exxon-Mobile, Alcoa).
(b) (10 Points) Here is another excerpt.

So which system will have the largest impact on carbon consumption? A 10% flat carbon tax might reduce the demand for carbon about 5% or less, according to an analysis by the Carbon Tax Center, an environmental advocacy group. That may not be enough. Businesses and governments haven’t figured out how the two competing regimes can work together, but in the end, the world may need both.

This quote seems to suggest that if the elasticity of demand for carbon is too low then a carbon tax would have to be supplemented with a cap-n-trade quota system to be effective. Comment on this.

**Answer.** Again consider any demand curve for carbon (See previous part for what I mean by this) and any quota amount $Q_{\text{target}}$ set in a CNT system. Let $P_{\text{quota}}$ be the price for permits after the quota is set and enforced. This must be the marginal willingness to pay for carbon at $Q_{\text{target}}$. Therefore the exact same quantity could be achieved by setting a CO$_2$ tax equal to $P_{\text{quota}}$. The degree to which the demand curve is inelastic does not affect this result at all! If the demand curve is very inelastic, then for a given reduction in CO$_2$ emissions we would have to set a very high tax, but that just means that the price of CO$_2$ in a CNT system would be *equally* high. **Therefore, the notion that that tax would have to supplemented with a quota in the event of inelastic demand is patently absurd.** The elasticity of demand for carbon is an interesting question, but *not* because answering it will favor either system over the other! A very very inelastic demand curve would be bad news for both approaches. Also the potentially lopsided distributional effects of a CNT system favoring carbon-intensive firm owners over all other households (discussed in the previous part) would be exacerbated in the case of very inelastic demand. Conversely, such Exxon stock-holding households may be even more disproportionately harmed under a CO$_2$ tax.

What if demand is “perfectly inelastic”? There are two issues here. First is *perfect* inelasticity possible? Second, assuming it is, could a CNT system offer something not possible with a mere tax? Let’s take the second question first. If demand is truly *perfectly* inelastic over all price ranges, it means that no matter high we set the price on carbon, firms will demand the exact same quantity. A quota would *legally* set the quantity lower, but how could it be enforced against perfectly inelastic demand? Its hard to argue that it could be enforced with the threat of fines. If it could, then we just need to set the tax at the level of the fines which would contradict the assumption that demand is perfectly inelastic. (Fines are just prices with a different name!) Then in order to argue that a quota could be effective in such a demand environment, one would have to argue that while firms are willing to pay an infinite amount in monetary sacrifices to keep their consumption of carbon the same, they are unwilling to make sacrifices along other dimensions - perhaps prison terms or executions for executives. Except that if their demand is truly perfectly inelastic, then they should be willing to spend an infinite amount protecting their executives from prosecution so they can keep consuming carbon at the same level (perhaps by raising their own army to fend off federal agents). Finally, if a quota is set on a good with such perfectly inelastic demand, then what would be
the price of the permits? It must be infinity! But this would mean that the price of all goods and services whose production uses carbon as an inputs must also be infinity. But such consumer prices would only be possible if demand for all of those good is also perfectly inelastic.

This brings us back to the question of whether perfectly inelastic is a real-world possibility. The answer has to be no. Why? The discussion aboves hints at the answer. Eventually, at a high enough price level, income effects must begin to kick in. To convince your self of this, think of some good - perhaps gasoline - for which you demand may be relatively insensitive to price. Suppose for example that if the price of gasoline rose from $2.50 per gallon $3.00 per gallon, your demand would not change at all. You have perfectly inelastic demand over that range of prices. But what if the price were $10 per gallon?, $1000?, $1,000,000? At some high enough level of price, your demand must decrease, beause your budget simply will not allow for it to remain the same.
10. (20 Points). Suppose that Allen has preferences for beer, \( b \), and other goods \( z \) according to the following utility function.

\[
u(b, z) = b^{1/20} z^{19/20}\]

Allen’s income is $1000 per month. The price of beer is $2 per pint.

(a) (3 Points) How much beer does Allen consume each month? **ANSWER** Recognizing the Cobb-Douglas preference represented by this utility function we can directly write down the characteristic constant income share demand function:

\[
b(p_b, p_z, m) = \frac{m}{20p_b}\]

hence for \( m = 1000 \) and \( p_b = $2 \) per pint, we have demand equal to \( b_0 = \frac{1000}{20 \times 2} \) (or 25).

(b) (3 Points) Suppose the price of beer increases from $2 to $2.50 per pint. What is Allen’s new demand for beer. **ANSWER.** Using the same demand function, we have \( b_1 = \frac{1000}{20 \times 2.5} \) (or 20).

(c) (14 Points) What part of the change in demand for beer due to a substitution effect? **ANSWER** As is usual with such problems, it will be helpful to find the compensated income level. Since the price of beer went up by $0.50 per pint and he was consuming 25 pints before the price increase, his income would have to be increased by \( .5 \times b_0 \) in order to continue to consume exactly the same bundle of goods he was consuming before the price increase. Hence the compensated level of income is \( m_c = $1000 + .5 \times b_0. \) (or 1012.50). Therefore the compensated demand is \( b_c = b(2.5, m_c) = \frac{m_c}{20 \times 2.5}. \) The substitution effect is the difference between this and the original demand. All in all we have

\[
SE = b_c - b_0
= \frac{m_c}{20 \times 2.5} - \frac{1000}{20 \times 2}
= \frac{1000}{20 \times 2}
= \frac{1000}{20 \times 2}
= \frac{1000}{20 \times 2}
\]

If you bothered to do the calculation (not required), you would have found that the compensated demand is 20.25 pints which means that the SE is -4.75 pints or almost the entire total effect of -5 pints. Intuitively, income effects will typically explain very little of the change in demand for a good when that good makes up a small portion of the consumer’s budget. This example confirms this intuition, though in general, it depends on the preferences. For example, if the consumer had Leontief preferences for pizza and beer, the entire decrease in demand would be explained by income effects.