

MIDTERM ANSWER KEY

ECON 210
PROFESSOR GUSE

Instructions. You have 2 hours to complete the exam. There are a total of 75 points available. It is designed to take about 1 minute per point. You are allowed to reference a single page of notes, 2-sided. You may *not* use any other notes, books or aids of any kind, be they human, electronic or mechanical. Calculations may be left in expression form for full credit. There is space provided for each question. If you need additional space, you may write on the back of the pages or use additional sheets and staple them to your exam when you turn it in. Please show all of your work. (Or at least enough so that the grader can figure out how you arrived at your answers.) Please write your name on the exam itself and record the time you started and time you finished. Finally please turn in your cheat sheet with your exam.

(1) SHORT ANSWER (20 Points)

- (a) (4 points) Consider the binary relation defined on the set of Winter 07 Intermediate Micro students, “sits to the left of”, as in “Student A [sits] / [does not sit] to the left of Student B.” Is this a *rational* binary relation? Explain why or why not. **ANSWER.** No, because it is not complete. Let $A \triangleleft B$ stand for “A sits to the left of B”. In order for \triangleleft to be complete we must have one or both of the following holding true for any pair of students, A and B .

$$A \triangleleft B$$

$$B \triangleleft A$$

Now consider two students, *Sally* and *John* who sit in different rows one directly behind the other. In this case, Sally $\not\triangleleft$ John and John $\not\triangleleft$ Sally. Hence \triangleleft is not complete. Note that if you interpreted “to the left of” in the weak sense of “sits at least as close to the left wall as”, than you could have answered that the relation is complete and therefore rational. Some of you answered that “to the left of” in not rational due to not being transitive. This is OK as long as you explain how a transitivity violation might come about (e.g. students sitting in a ring) *Grading Note* Several people answered that because the relation is binary, it must be complete. This is not true. You confusion

stems from a common interpretation of completeness as “always has an answer”. Well it is true binary relation always has an answer for any ordered pair of element from the set on which it is defined. But that is not the definition of completeness. Completeness says that the answer will be YES (i.e. TRUE) on an ordered pair if the answer was NO when the order was reversed. In the context of preferences “always has an answer” or “always able to rank” is common way to express this idea, but unfortunately not a very precise one.

- (b) (3 points) Harold has some potatoes and some fried chicken. He can exchange potatoes for fried chicken at the rate of 3 pounds per bird. He is willing to give up potatoes for fried chicken at the rate 2 pounds per bird. Recommend a feasible trade for Harold to improve his welfare? If there isn't one, state why. **ANSWER.** Harold is willing to give up fewer potatoes for chicken than he has to. Put another way, he willing to give up more chicken for more potatoes than he has to. Therefore he should trade some of his chicken for potatoes at the going rate to make himself better off.
- (c) (3 points) Write down a condition involving a discount rate r which, if true, suggests that building a dam which costs \$50 million to build and will provide \$2 million per year in benefits forever is worthwhile. **ANSWER.** The net present value (in millions) of the project can be expressed as

$$NPV = -50 + \frac{2}{r}$$

Where -50 million is the cost of the project (assumed here to be paid up front) and $\frac{2}{r}$ million is the present value of the perpetual benefit stream. The project is worthwhile whenever $NPV > 0$ or $\frac{2}{r} > 50$ or $r < \frac{2}{50}$ (4 %).

- (d) (4 Points) In an apparent contradiction to the law of demand, under what conditions could an individual's demand for a normal good increase after an increase in the price of that good? **ANSWER** This exact question was asked in Homework # 4, Question # 3, part (d) in the context of individual labor supply. In general the answer is that such an effect can happen if the income effect is positive. For a normal good, this is only possible when the individual *owns* the good whose price increased. More specifically, the individually must be a net supplier of the good in question and the income effect must be large enough that it overcomes the substitution effect (which must be negative, by the law of compensated demand).
- (e) (6 points) Carrie borrowed \$20,000 for a new car at an annual rate of 8% interest. She will make annual payments of \$3000 starting in exactly one year. How long will

it take Carrie to pay off her loan? (Don't calculate; just write an expression.) **ANSWER** Using the formula for the present value of a finite stream of identical payments we have

$$20000 = \frac{3000}{.08} \left(1 - \frac{1}{(1.08)^T} \right)$$

where T is the number of years the finite stream lasts - or in this case, the number of years it will take Carrie to pay off the loan. *GRADING NOTE:* 5 out of 6 points for getting this far: setting it up and interpreting the equation properly.

We can get an explicit expression for the number of years by solving for T in the equation above.

$$\begin{aligned} 20000 &= \frac{3000}{.08} \left(1 - \frac{1}{(1.08)^T} \right) \\ \Leftrightarrow \frac{20000(.08)(1.08)^T}{3000} &= (1.08)^T - 1 \\ \Leftrightarrow (1.08)^T \left(1 - \frac{20000(.08)}{3000} \right) &= 1 \\ \Leftrightarrow T \log(1.08) + \log \left(1 - \frac{20000(.08)}{3000} \right) &= 0 \\ \Leftrightarrow T &= - \frac{\log \left(1 - \frac{20000(.08)}{3000} \right)}{\log 1.08} \end{aligned}$$

The numerical answer (which you were not expected to calculate) turns out to be 9.9 years. (Annual payments of \$2980.59 would make it exactly a ten year loan at 8%. Final exam question: How would you figure that out?)

- (2) (15 Points) Sally has smooth convex rational preferences for consumption and leisure. Assume that both leisure and consumption are normal goods. Sally has a very flexible employer who lets her work as many hours per week as she likes up to 60 and pays her \$5 per hour. Under the status quo (SQ), Sally is guaranteed government assistance of \$100 per week.
- (5 points) Draw Sally's budget under the status quo.
 - (1 points) Under the status quo, Sally chooses to work 20 hours per week. Mark this choice in your diagram and label it "SQ".
 - (5 points) Tad Chiggrin complains that the government should not have to pay for welfare benefits for people like Sally who earn their own income. He proposes welfare reform plan "C" where a welfare recipient's assistance is reduced by \$1 for each dollar that person earns and predicts that his proposal will remove people like Sally from the

welfare rolls saving the government millions of dollars. Draw Sally's budget under plan "C" in the same diagram.

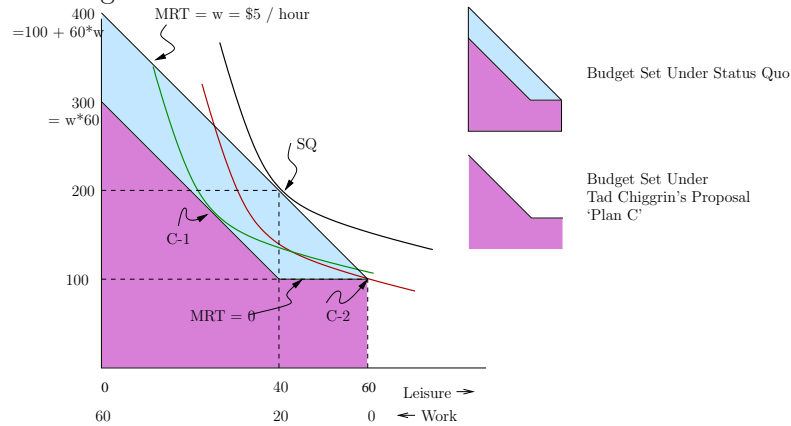


Figure 2. The only thing we know for sure about Sally's preferences is that 20 hours of work is an optimal choice under the status quo and that both goods - consumption and leisure - are normal. The black indifference curve along with the green one are meant to represent one possibility for Sally's preferences consistent with these assumptions. This "green" possibility suggests that Sally will increase the number of hours she works under Plan 'C'. The black indifference curve along with the red one are meant to represent another qualitatively different possibility. This "red" possibility suggests that Sally might actually decide to not work at all under Plan C. Both are plausible given the facts in the problem.

- (d) (4 points) A prominent liberal lawmaker Saul Goodrock says the Mr. Chiggrin's proposal will result in poverty for people like Sally and moreover plan 'C' is unlikely to save the government any money. Is Goodrock's critique valid? Explain. **ANSWER** Senator Goodrock could be exactly correct *if* Sally's preferences for consumption and leisure are similar to the those represented by the red indifference curve in the diagram. Since Sally's effective wage under plan 'C' is zero for the first 20 hours of work. It seems reasonable that she might lose all incentive to work. (See point 'C-2' in Figure 2.) In this case, her consumption would drop in half to \$100 and the government would save exactly nothing on her welfare checks. On the other hand, the truth could be closer to what Representative Chiggrin had in mind *if* Sally's preferences are more like those represented by the green indifference curve in the diagram. In this case, Sally, chooses to work more hours under plan 'C'. (See point 'C-1' in Figure 2.) That must mean more than 20 hours and so the government's cost in terms of welfare spending would drop to zero. However, because both goods are normal, we know that she would never work more than 40 hours per week under plan 'C' and therefore must be making a total of less than \$200 - her original income under the status quo. Perhaps, it is interesting, then, to note that the government will either save nothing on welfare spending

on Sally or will save the entire amount of \$100. Nothing in between is possible given our assumptions. However, if we consider that the population is likely to be filled with a variety of preferences - some more like the red Sally and some more like the green Sally - then total government spending will go down to something greater than zero and welfare recipients will all be worse off. Possible Final Exam Question: How does the Earned Income Tax Credit address competing concerns in the welfare debate of providing incentive to work while relieving poverty? Is the EITC the best solution you can think of?

- (3) (20 Points) Fanny has preferences for this year's consumption (c_1) and next year's (c_2) represented by the following utility function.

$$u(c_1, c_2) = \log c_1 + \log c_2$$

Fanny expects to earn \$500 in both periods. Fanny can save and borrow at an interest rate of 25%.

- (a) (4 points) Draw Fanny's budget set. Be sure to label the intercepts and indicate the MRT. **ANSWER.** See Figure 1.
- (b) (4 points) Sketch the indifference curve that runs through her endowment. What is Fanny's MRS at her endowment point? Be sure to interpret your answer. **ANSWER.** The MRS measured at (500, 500) is given by

$$\begin{aligned} MRS(500, 500) &= \frac{\partial u(500, 500)/\partial c_1}{\partial u(500, 500)/\partial c_2} \\ &= \frac{1/500}{1/500} \\ &= 1 \end{aligned}$$

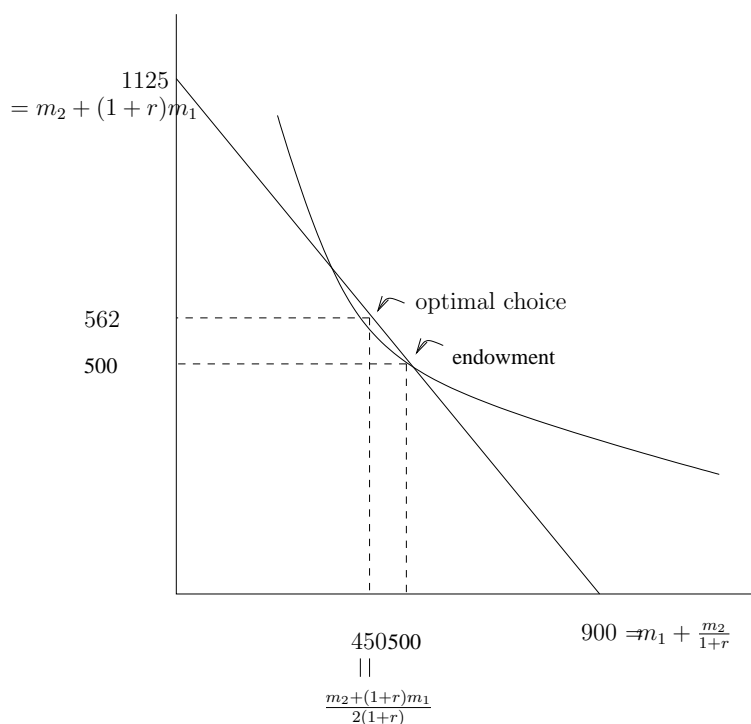


Figure 1. The MRS at the endowment point (500, 500) is exactly equal to 1. The MRT along the budget is $(1 + r)$ or 1.25. As such there is a portion of the budget line lying in the direction of more future consumption and less present-day consumption from the endowment point which represents an improvement over the endowment. The optimal choice must lie in there as shown.

- (c) (2 points) Will Fanny save or borrow? Explain. *ANSWER.* By the answers just given, we see that at her endowment, Fanny's MRS is equal to 1 which is less than the MRT which is equal to $(1 + r) = 1.25$. In words, Fanny is willing to give up less consumption tomorrow for another dollar in consumption today than she has to. Or put another way, she is willing to give up more consumption today for another dollar of consumption tomorrow than she has to. Therefore she will save - that is, spend less today than she earns. This conclusion is verified by inspecting the indifference curve running through her endowment point. Since the slope of the IC is less than that of her budget line, there is portion of her budget lies in the direction of less spending today and more tomorrow from her endowment point which lies above her endowment point w.r.t. preferences.
- (d) (6 points) Write down an expression in terms of the interest rate r which represents how much Fanny will save or borrow? (Hint. You may want to write down Fanny's demand for c_1 first.) *ANSWER.* The given utility function was Cobb Douglas with equal coefficients on the log terms which suggests equal "income shares". The challenge is deciding what "income" is. One could use present value of the endowment or future

value of the endowment for the income, it doesn't matter as long as one is consistent with how the price of each period consumption is expressed. I will use present value. Let $M = m_1 + \frac{m_2}{1+r}$. Since the income is expression in terms of present value, that means the "price" of today's consumption is 1, while tomorrow's consumption is $\frac{1}{1+r}$. Hence demand for present consumption, c_1 in terms of r is given by

$$\begin{aligned} c_1(r) &= \frac{M}{2} \\ &= \frac{m_1 + \frac{m_2}{1+r}}{2} \\ &= 250 + \frac{250}{1+r} \end{aligned}$$

Savings, $S(r)$ is the difference between the today's endowment and today's consumption

$$S(r) = m_1 - c_1(r) = 250 - \frac{250}{1+r}$$

Note that when $r = .25$, Fanny saves \$50.

- (e) (4 points) Would Fanny ever switch her saving/borrowing decision? How much and in what direction would the interest rate have to change? (Hint. You can answer this question even if you were unable to answer the previous one.) **ANSWER** Using the expression for savings just derived let us set find the rate of interest at which Fanny's savings would be less than zero. (i.e. switch to borrowing)

$$\begin{aligned} S(r) &< 0 \\ \iff 250 - \frac{250}{1+r} &< 0 \\ \iff r &< 0 \end{aligned}$$

In other words, the rate of interest, r , would have to drop all the way to zero in order for Fanny to save zero and would have to be negative in order for Fanny's optimal choice to involve borrowing. What if you were unable to derive an expression for savings? The hint suggests that you should still be able to answer this question. One can verify that the interest rate would have to be negative in order to get Fanny to borrow simply by looking at the diagram. Lowering the interest rate would rotate her budget line around her endowment point. If $r = 0$ then the budget line would have a slope of exactly 1 - the same as the indifference running through that point. If $r < 0$, the budget line would be "shallower" than her MRS and she would optimally

choose to borrow. (See explanation for part (c) replacing occurrences of “today” with “tomorrow” and vice-versa.)

- (4) (20 Points) Fred cares about the consumption of his two children Al and Betty according to the following utility function.

$$u(c_A, c_B) = \min \{c_A, c_B\}$$

Fred has allocated \$200 per week in allowance money for his children. Let t_A be the transfer Fred gives to Al and t_B the transfer Fred gives to Betty. In addition, Al and Betty both have jobs. Let m_A be what Al earns in his job and m_B what Betty earns, so that $c_A = m_A + t_A$ and $c_B = m_B + t_B$

- (a) (6 points) Suppose Al and Betty earn the same income. In particular, assume that $m_A = m_B = 50$. Draw Fred’s budget and label his optimal choice. Describe his choice in terms of the allowance he gives to each of his children.

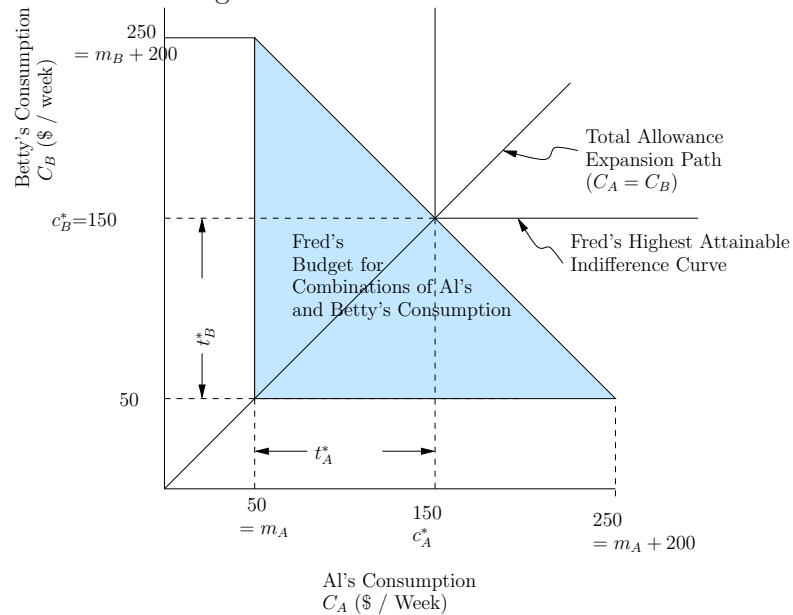


Figure 3. Fred’s Budget for combinations of his childrens’ consumption levels is depicted by the blue shaded area. The expansion path marked in the diagram describes Fred’s choices for all levels of total spending on his children. It lies on the 45 degree line because of his perfect complement preferences. His optimal choice (c_A^*, c_B^*) is $(150, 150)$ as shown in the diagram. In terms of transfers, this amounts to \$100 for each child. The extra fifty they each spend comes from their own earnings. Note that Fred’s budget does not include the origin or the neighborhood around it in this diagram - just the blue area. This is because I assume that Fred cannot confiscate his childrens’ earnings; he can only give them money.

- (b) (6 points) Suppose Al earns more than Betty. In particular, assume that $m_A = \$100$ and $m_B = 50$. Redraw Fred's Budget (in a new diagram, if you like) and label his optimal choice. How does Fred allocate his childrens' allowances now?

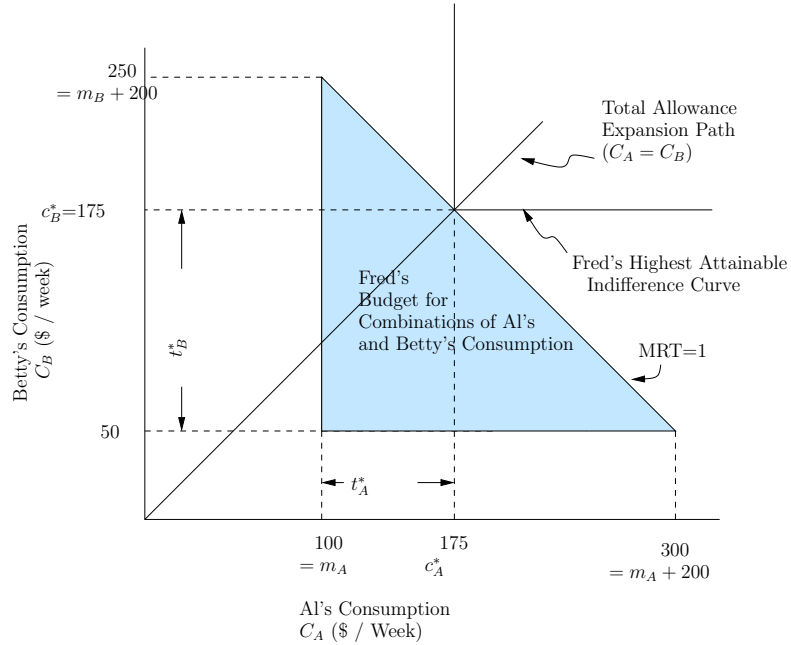


Figure 4. The case where Al earns \$100 and Betty earns just \$50. Again, Fred's Budget for combinations of his childrens' consumption levels is depicted by the blue shaded area. The optimal choice in terms of his childrens final consumption levels is $(c_A^*, c_B^*) = (175, 175)$. This amounts to giving Al \$75 while Betty get \$125. It is worth noting that when Al's income increased by \$50 his share of the allowance decreased by \$25. His effective wage would appear to be half of his nominal wage. More about this in the next question.

- (c) (8 Points) Fix Betty's income at \$50 per week and assume that Al earns \$10 per hour. Consider Al's labor supply decision. Draw his budget for leisure and consumption. Assume that Al could work up to 40 hours per week. How does his wage compare to the marginal rate of transformation he faces?

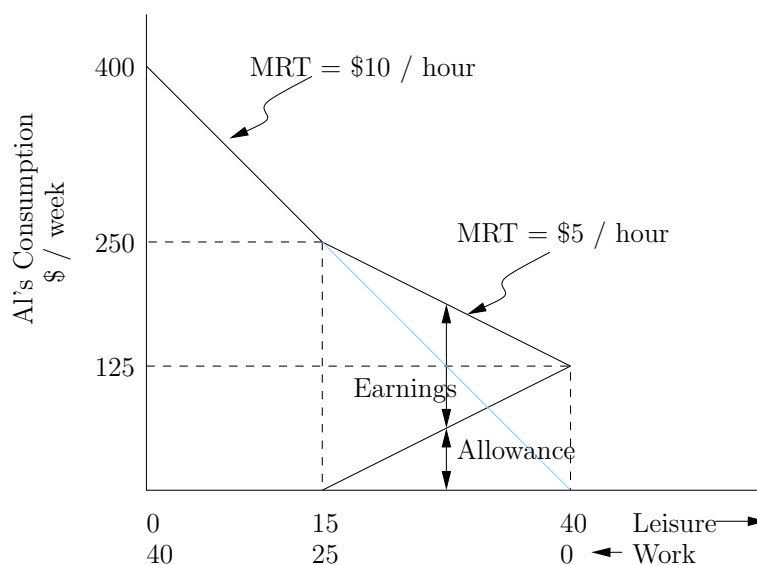


Figure 5. Al's budget for leisure and consumption given Fred's preferences for his children and Betty's choice to earn \$50 per week.

ANSWER. See Figure 5. Note that when Al does not work at all (40 hours of leisure), his consumption level would be \$125. This is because he effectively splits with his sister the \$50 she earns and the \$200 his father has allocated to his children, which is to say his father would choose to give Betty \$75 and Al \$125 to make their consumptions equal. At first each hour that Al works, increases his consumption by just \$5 - instead of the \$10 he earns. Why? Because each dollar he earns reduces his allowance by \$0.50 since his father will adjust to keep the two children's consumption equal. However, if Al works 25 hours or more, his allowance will be reduced to zero and so cannot be reduced any further, which means that the MRT will increase to \$10 per hour - his wage. Possible Final Exam Question: Explain how Fred's allowance affect Al's labor supply decision. Be sure to discuss income and substitution effects. Assume the leisure and consumption are normal.