

MIDTERM EXAM ANSWERS

ECON 210
PROFESSOR GUSE

Instructions. You have 3 hours to complete the exam. There are a total of 75 points on the exam. The exam is designed to take about 1 minute per point. You are allowed to reference a single page of notes, 2-sided. You may *not* use any other notes, books or aids of any kind, be they human, electronic or mechanical. Calculations may be left in expression form for full credit. There is space provided for each question. If you need additional space, you may write on the back of the pages. Please justify and explain your answers where needed and show your work. (Or at least enough so that the grader can figure out how you arrived at your answers.) Please write your name on the exam itself and record the time you started and time you finished. Finally, please turn in your cheat sheet with your exam.

(1) SHORT ANSWER (25 Points)

- (a) (5 points) $(x_1, y_1) \sim (x_2, y_2)$ while $(x_1, y_1) \succ \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Which standard assumption on consumer preferences does \succsim violate? Briefly explain. **ANSWER.** By definition a rational preference relation \succsim is *convex* if whenever you have two consumption bundles on the same indifference curve, e.g. $(x_1, y_1) \sim (x_2, y_2)$ any weighted average of those two should be at least as well as either of them. In other words, for all $\alpha \in (0, 1)$ if *convexity holds*, we should have $(\alpha x_1 + (1 - \alpha)x_2, \alpha y_1 + (1 - \alpha)y_2) \succsim (x_1, y_1)$. However, we see that this is not true from $\alpha = \frac{1}{2}$. Therefore convexity does not hold.
- (b) (5 points) McBarry faces constant prices for hockey pucks, minivans and Nascar tickets. He must give up 10 hockey pucks for each minivan he buys and half of a minivan for each Nascar ticket. At what rate must he give up hockey pucks for Nascar tickets? **ANSWER** Let

- P_{mv} be the price of minivans.
- P_{hp} be the price of hockey pucks.
- P_{nc} be the price of nascar tickets.

“He must give up 10 hockey pucks for each minivan” means that $\frac{P_{mv}}{P_{hp}} = 10$ while “half of a minivan for each Nascar ticket.” means that $\frac{P_{nc}}{P_{mv}} = \frac{1}{2}$. We are looking for $\frac{P_{nc}}{P_{hp}}$ which must be equal to $\frac{P_{mv}}{P_{hp}} \frac{P_{nc}}{P_{mv}}$, so $10 \cdot \frac{1}{2}$ or 5.

GRADING NOTE. I gave full credit to answer that came up with 5 by some reasonably logical process. However, the point here is that price ratios represent MRTs. So if you didn’t take a price ratio approach, please be sure you understand why this works before the final.

- (c) (3 points) If $u(x_1, x_2)$ represents Molly’s preferences for goods 1 and 2, construct another utility function which does just as good a job of representing Molly’s preferences. **ANSWER.** Any other function v such that $v(x_1, x_2) = f(u(x_1, x_2))$ and where f is a strictly increasing function. For example any of the following would be acceptable answers.

- (i) $v(x_1, x_2) = \log u(x_1, x_2)$ ¹
- (ii) $v(x_1, x_2) = e^{u(x_1, x_2)}$
- (iii) $v(x_1, x_2) = 3657^{u(x_1, x_2)}$
- (iv) $v(x_1, x_2) = 89 + 4.3u(x_1, x_2)$

GRADING NOTE Several of you gave answers along the lines of $v(x_1, x_2) = u(2x_1, 2x_2)$. This will not always result in a monotonic transformation. Instead of a guaranteed order-preserving relabeling of indifference curves, this would relabel the utility of each bundle with the number assigned by u to the bundle with twice as much of each good - a subtle but important difference. If preferences happen to be *homothetic* then, in fact, this does result in a monotonic transformation. In fact the definition of homothetic preferences is that they can be represented by a utility function u such that $u(2x_1, 2x_2) = 2u(x_1, x_2)$. But homotheticity is a special case.²

- (d) (3 Points) Assume the decision maker can borrow and lend at rate r . The present value of a \$200 benefit in 4 years followed by a \$300 cost in 7 years is equal to

¹Or more precisely, if negative number are in the range of u and since \log is defined for negative number a safer log transform might look more like $v(x_1, x_2) = \log(u(x_1, x_2) + 1 - \min u(x_1, x_2))$

²Granted, the three big ones, (i) perfect substitutes, (ii) perfect complements and (iii) cobb-douglas are examples of homothetic preferences. However, quasilinear preferences, in general are not homothetic.

----- **ANSWER.**

$$NPV = \frac{200}{(1+r)^4} - \frac{300}{(1+r)^7}$$

- (e) (3 Points) Write down a utility function for some one who always spends $\frac{1}{3}$ of his income on cheeseburgers and $\frac{2}{3}$ of his income on everything else. **ANSWER.** Any monotonic transformation of $u(C, E) = C^{\frac{1}{3}}E^{\frac{2}{3}}$ where C is the cheeseburger consumption level and E is the consumption level of everything else. This is what we mean when we say that Cobb Douglas preferences lead to constant income share demand functions.
- (f) (3 Points) Write down a utility function for some one who consumes water and other things. Make it so that their demand for water has an income elasticity of zero for high enough income levels above a certain threshold and income elasticity of 1 for income below that threshold. **ANSWER.** The right answer has to represent quasilinear preferences since that is how you get the income expansion path described in the problem. For example $u(W, X) = X - (W - 100)^2$, where W is the consumption of water. In general $u(W, X) = X + f(W)$, where f is some strictly concave function works. See answer key to Homework 3, Question 4 for a more complete explanation. **GRADING NOTE:** Note that $u(W, X) = X + \beta W$ (perfect substitute preferences) is an example by most definitions of quasilinear preferences. However, in this case, there is no price ratio that would generate an income expansion path fitting the story described in the problem. For prices of water above β , a consumer with these preferences would have a demand of zero water at all income levels (so zero income elasticity at all income levels). For prices of water below β , this consumer would have an income elasticity of 1 at all income levels. If the price of water were exactly β then any choice on the budget line is a solution and the income expansions path (and by extension income elasticity) can be pretty much anything you like including what is described, but also exactly the opposite. I awarded one point to answers with perfect substitutes preferences.
- (g) (3 Points) Low interest rates tend to favor projects with what kind of time structure of costs and benefits? **ANSWER.** projects with upfront costs and delayed benefits tend to stand a better chance when interest rates (opportunity

cost of capital) is low. This was demonstrated in the Present Value questions, parts (a) and (b) on homework 4.

- (2) (15 Points) Cookie Monster (CM) has nice rational preferences for cookies and milk. The price of cookies is \$1 each. The price of milk is \$1 per glass. When Cookie Monster (CM) is given \$300 in cash, he chooses to buy 250 cookies. On the other hand, if you give CM \$200 in cash and \$100 in milk vouchers, he spends all of his cash on cookies and drinks 100 glass of milk. In this case, what can you say about CM's *marginal rate of substitution* of milk for cookies? In other words, what is CM's MRS at the point (cookies, milk) = (200,100)? Explain using a diagram. Be sure to interpret your answer. **ANSWER.** Note that the point $(x_C, x_M) = (200, 100)$ is *on* Cookie Monster's original cash budget line. Since he chose point (250, 50) when he faced that budget line and he has convex (nice) preferences, it follows that "uphill" along that budget line from point (200, 100) is toward point (250, 50). In other words, from that point, CM must be willing to give up *more* milk for cookies than he has to when he faces the cash budget. The MRT under the cash budget line was $\frac{p_C}{p_M} = 1$. Therefore, from that point he is willing to give up *at least* one glass of milk to get another cookie. We cannot really say how much greater than 1 his MRS is at that point. See Figure 1.

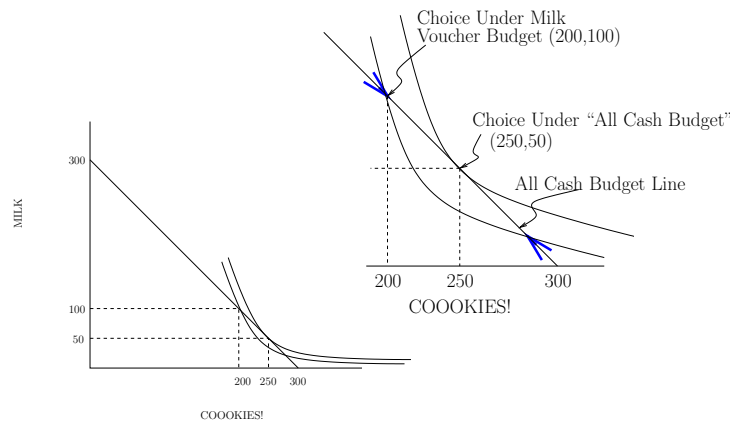


Figure 1. In the lower left we have a sketch of CM's choice when faced with the all cash budget. The diagram in the upper right is a detail. Note that since CM chooses the point $(250, 50)$ when given the cash budget, that must be the highest point (in preference) on that entire budget line. Therefore from any other point on that budget line, uphill be toward that point. In the detail figure, the blue arrows indicate which way is up. The indifference curves drawn here are merely meant to be consistent with the assumption of convex preferences and the choices observed. Other shapes are possible.

- (3) (15 Points) Sally only eats cheese and corn in the form of home-made cheetos - which she make herself from a very exacting recipe. Hence she has *perfect complements* preferences for cheese and corn. Briefly describe the income and substitution effects of a price decrease in corn.

The total effect on Corn is [positive].

The total effect on Cheese is [positive].

The substitution effect on Corn is [zero] .

The substitution effect on Cheese is [zero] .

The income effect on Corn is [positive].

The income effect on Cheese is [positive].

Explain using a diagram.

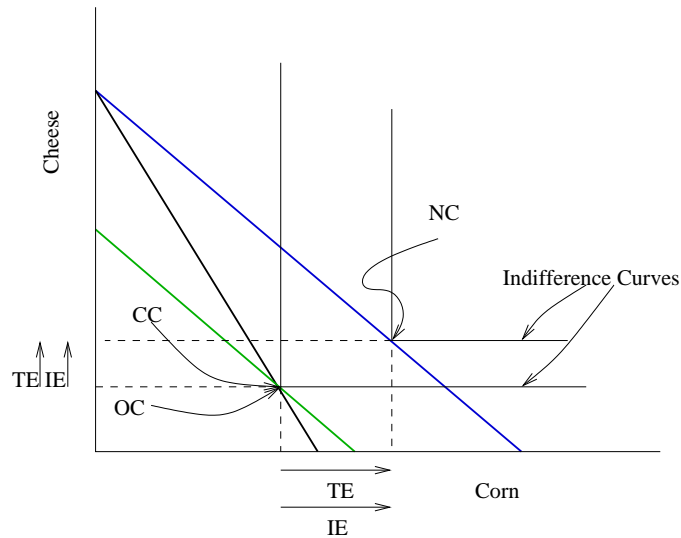


Figure 2. A price decrease in Corn. The black budget line is the old budget line. The Blue one is the new. The Gree Line is a compensated Budget Line. “OC” is the original optimal choice. “NC” is the new optimal choice after the price decrease. “CC” is the compensated Choice.

Note that the compensated choice (CC) and the old choice (OC) are exactly the same. This is a consequence of perfect complement preferences. Since the substitution effect is defined as the difference between the CC and the OC, the substitution effect on the demand for both good here is exactly zero. Therefore the entire total effect is explained by the income effect - which is positive.

- (4) (20 Points) Joe expects to live for two periods. Joe has current income equal to \$2000 and will receive income in the next period equal to \$1000.
- (a) (2 Points) If the interest rate were $r_0 = .25$, he would save \$400. How much will Joe have to consume in the next period? **ANSWER.** In general if (m_1, m_2) represent the endowment point (income path), (c_1, c_2) the consumption path and s stands for savings ($s = m_1 - c_1$), then we must have

$$c_2 = m_2 + s(1 + r)$$

So in this case $c_2 = 1000 + 400(1.25)$ or \$1500.

- (b) (2 Points) If the interest rate were $r_1 = .10$, he would save only \$200. How much will Joe have to consume in the next period? **ANSWER** By the same logic as above, $c_2 = 1000 + 200(1.1)$ or \$1220.

(c) (8 Points) If we think of $r_0 = .25$ as the original interest rate, which effect dominates along the current consumption axis as we lower the interest rate to $r_1 = .10$? Substitution Effect or Income Effect ? Explain using a diagram.

ANSWER. Your diagram must show a positive substitution effect and negative income effect along the current consumption axis. Why? Each effect in turn...

- $SE_0 > 0$. The positive SE follows from the fact that an interest rate decrease lowers the MRT. That is when interest rates are lower, less future consumption must be forgone in order to increase current consumption. Therefore by the law of compensated demand, the SE must be in the direction of increased current consumption and decreased future consumption.
- $IE_0 < 0$. Since this consumer would be a saver at the higher rate, the new budget line represents a lower overall wealth level when compared to the Slutsky compensated budget line.³ Since current consumption is a normal good, the IE must therefore be negative along both axes.

Since we are told that current consumption would be higher (savings lower) under the lower rate, this means that the TE along the current consumption axis is positive - in the same direction as the SE. In other words, the SE dominates. Note that along the future consumption axis, the two effects work in the same direction.

³The current consumption Slutsky IE will *always* be negative for an interest rate decrease for someone who was a saver at the higher rate. However, the Hicksian income effect for an interest rate decrease *could* be positive for a saver, but this is a possibility only if the new choice involves borrowing which is not the case here.

