Instructions. You have 3 hours to complete the exam. You will answer questions worth a total of 100 points. Please write your responses on the exam itself in the space provided. If you need additional space, there is blank sheet included at the end. You may refer only to your own handwritten, “cheat sheet”. Calculators and all other references materials are not allowed. If a question asks for a numeric quantity you may leave your answer in expression form for full credit. (e.g. \( \frac{40-30}{5} \) would be perfectly acceptable in place of “2”.) Be sure to label any diagrams you draw, to show your work and to explain your reasoning. Finally, take note that questions are printed on BOTH sides of each page. You may keep your cheat sheets. Thank you and good luck!

Name:

Pledge:
1. [5 Points] List two of the main findings from the article by Card and Dahl (2009) on football and domestic violence that we discussed on the last day of class. **Answer** Here are the four points which I labeled as “main findings”

(a) Home town NFL “upset losses” increase male-on-female domestic violence rates by approximately 8% on average in the sample. An upset loss is defined as an event where the team was predicted to win by more than 3 points, but in fact, ended up losing.

(b) On average “upset wins” do not have any complementary protective effect - except perhaps in the case of 4pm Sunday games.

(c) Female-on-male domestic violence rates are unaffected by NFL game outcomes.

(d) Salience and Frustration exacerbate the effects of upset losses on violence rates - in some cases doubling the effect. The authors measure salience with playoff contention and whether the home team is playing against a traditional rival. Frustration is measured by whether or not there were excessive penalties, turnovers or sacks in the game.

**Grading**. Any two of these two obviously be acceptable for full credit. If you list some other valid point or claim from the paper that would be fine too. 3 points for listing one legit finding, 2 for the second. Vague descriptions of the findings get 1 point. For example, “televised football games lead to domestic violence” would be a 1 point answer.

2. [3 Points] Define conditional factor demands. **Answer** For a given quantity of output $y$, the conditional factor demands are the choices of inputs which minimize the cost of producing that level of output. Therefore, besides typically depending on factor prices, CFDs also depend on the output choice $y$. Furthermore, unlike factor demands (which represent the solution to the full profit maximization problem), CFDs do not depend on the price of the firm’s output.

3. [2 Points] What is the relationship between conditional factor demands and the cost function? **Answer** The cost function is simply the cost of the CFDs - that is the dot product of the CFD and their corresponding factor prices. For example, the two-good case, if $x_L(y, w_L, w_K)$ and $x_K(y, w_L, w_K)$ are the CDF for labor ($L$) and capital ($K$) then the cost function is

$$c(y, w_L, w_K) = w_L x_L(y, w_L, w_K) + w_K x_K(y, w_L, w_K)$$
4. [ 2 Points ] In order for a natural monopoly to develop, it 

(a) is important that the firm be very large.
(b) is important that the firm prices its product below cost.
(c) must have large fixed costs relative to the total market demand.
(d) must be in the presence of government intervention.

The key ingredient for a natural monopoly is that the technology tends to exhibit decreasing average cost over all or nearly all the range of demand. Hence (c) is clearly the best answer.

5. [ 3 points ] For a single price monopoly, the profit maximizing price will be _____ than marginal revenue and _____ than marginal cost.

(a) greater; greater
(b) greater; less
(c) less; greater
(d) less; less

First note that at the profit maximizing solution, it means producing a quantity where marginal revenue is equal to marginal cost. That rules out (b) and (c). Next, recall that the monopolist’s choice can be characterize as choosing a point on the demand curve and the marginal revenue is everywhere below the demand curve. Hence (a) is clearly the best answer.
Webco can produce websites with labor \((l)\) and two types of capital - computers \((c)\) and office space \((s)\) - according to the following production function.

\[
y = \min\{x_c, x_s, x_l\}
\]

Factor prices are \(w_c = 10\), \(w_s = 10\) and \(w_l = 10\). For each of the following situations, what is the minimum price at which Webco is willing to produce more than zero output. Assume the standard competitive market conditions. **PREFACE TO ANSWERS** In general, any cost-minimizing approach to production with this technology will always involve all three inputs in equal proportion. Moreover, if one or more inputs is fixed in the short run, then output expansion is limited to augmenting the other inputs and only if the other inputs are present is lesser amounts than the fixed input. This is due to the perfect complementarities (no substitution).

(a) Webco has committed to renting 5 units of office space and 5 computers, but has not committed to hiring any labor. **ANSWER** Each unit of labor they hire will increase output by one unit up to five units. For example with no labor their output would be \(\min\{5, 5, 0\} = 0\), while hiring one unit would yield \(\min\{5, 5, 1\} = 1\). Since each unit of labor is $10, marginal costs are a constant $10 per unit for the first five units. Therefore the answer to the question is $10. To increase output over five units, the firm would need to augment all the inputs. Hence marginal cost would increase $30 after 5 units.

(b) Webco has committed to renting 15 units of office space, but has no computers and has not committed to hiring any labor. **ANSWER** By similar logic as above, the firm need to hire one unit of labor and one computer for each unit of output up to 15. Therefore the marginal cost of output is $20 for the first 15 units. This is also the threshold price.

(c) Webco has not committed to hiring any inputs. **ANSWER** In this case, the firm’s marginal cost $30 starting with the very first unit of output. Hence $30 is the threshold price.

**Grading Note** 5 points awarded for getting at least one part right and recognizing that the threshold price in any case depends on what is variable. Otherwise 3 points for each right answer when only one or two answers are correct. 2 Points for writing that they must cover variable cost. 5 points for a general recognition that this is analogous to melon farmer problem and that threshold price increase as the number of commitments goes down.
7. [ 10 Points ]. Assume that both leisure and consumption are normal goods. Explain why an individual labor supply curve may be “backward-bending” - that is, portions where labor supply is decreasing in wage. Your explanation should include a diagram with budget lines for leisure and consumption. ANSWER See Homework #4, Problem #3.

Grading Note 5 points for explanation and 5 points for diagram.

8. [ 15 Points ] A profit maximizing firm faces a wage for labor of $20 per hour and capital price of $40 per unit. It produces output according to

\[ y = L^{\frac{1}{2}} K^{\frac{1}{4}} \]

(a) At the optimal choice of \( L \) and \( K \), what is \( \frac{K}{L} \)? ANSWER At the cost-minimizing choice we must have MRtS equal to the ratio of the input prices.

\[ \frac{\partial f}{\partial L} = \frac{w_L}{w_K} \Rightarrow \frac{2K}{L} = \frac{1}{2} \]

\[ \Rightarrow \frac{K}{L} = \frac{1}{4} \]

In words, at the given input prices, the firm will always employ four times as much labor as capital.

(b) Derive the firm’s cost function. ANSWER Besides the first order condition (previous part) the firm must also produce where \( L^{\frac{1}{2}} K^{\frac{1}{4}} = y \) where \( y \) is the target level of output from the cost minimization problem. Using the restriction that \( \frac{K}{L} = \frac{1}{4} \) we can substitute in \( 4K \) for \( L \) to get

\[ (4K)^{\frac{1}{2}} K^{\frac{1}{4}} = y \Rightarrow 2K^{\frac{5}{4}} = y \]

\[ \Rightarrow K = \left( \frac{y}{2} \right)^{\frac{4}{3}} \]

So that is our conditional factor demand for capital. We already know that the firm will four times as much labor so the CFD for \( L \) is

\[ L = 4 \left( \frac{y}{2} \right)^{\frac{4}{3}} \]

Hence the cost function (at the given input prices) is given by

\[ c(y) = (4 \cdot 20 + 40) \left( \frac{y}{2} \right)^{\frac{4}{3}} \]
Grading Note 7 Points for part (a), 8 Points for part (b). In part (a) 2 Points for simplifying the MRTS, 5 Points for setting it equal to the correct price ratio. 1 Points for setting it equal to the upside down price ratio. In part (b) 3 points for the idea of making the substitution back into the production function and trying the solve for the CFDs; 4 Points for actually executing this idea correctly. Another 3 Points for the idea of taking the CFDs to build the cost function. 4 Points for actually executing that part correctly as well.

9. [ 25 Points ] Suppose that each unit of some kind of pollution always causes $4 in damages (constant marginal damage). Answer each of the following questions using diagrams if needed.

(a) [ 5 Points ] Suppose that demand for the pollution is given by $Q = 10 - P$. What is the efficient level of pollution? ANSWER The efficient level of pollution is where marginal willingness to pay for it is equal to the marginal damage it causes. We can re-write the demand function as a MWTP function by solving for $P$: $MWTP(Q) = 10 - Q$. Setting this equal to $MD$ we get

$$MWTP(Q_{eff}) = MD$$

$$10 - Q_{eff} = 4$$

$$Q_{eff} = 6$$

Grading Note Couldn’t be simpler; 5 or nothing.

(b) [ 10 Points ] Describe the welfare effects of achieving the efficient level of pollution with a tax versus a system of tradeable permits. Be sure to describe the effect on polluters, the effect on those suffering the damages, and the effect on government revenue (or other taxes). Does the allocation of permits make a difference for any of these groups? ANSWER An efficient tax would be set to marginal marginal damages or $4 per unit. An efficient quota would be set to the efficient level of pollution or 6 units. It does not matter which system is implemented. The welfare effects on all group can be made identical by adjusting how tax revenue is distributed in the case of a tax or how the tradable permits are initially allocated in the case of a quota.

Let’s assume that in both cases rights are allocated to income tax payers. The effect of either system on polluters is a welfare loss equal to the area to the left of the demand curve between 0 and $4 per unit. The effect on those suffering the damages is a welfare gain equal to the area below the marginal damage curve between 6 and 10 units of
pollution. The net effect on government revenue is zero while the gain for income tax payer is $24. This is because in the case of the tax, $24 is collected in tax revenue and then redistributed to income tax payers. In the case of the tradable permit system, income tax payers get the initial allotment of permits and sell them for $4 each, gaining $24.

**Grading Note** 2 points each for effect on (i) polluters (ii) damages and (iii) revenue. 4 Points for discussion of permit / revenue allocation. I awarded credit for consistent answers even if diagram was based on entirely erroneous answer from part (a). Still diagrams had to be well-labeled and connected to explanations.
(c) [10 Points] Now suppose that demand for the pollution is uncertain. With probability .5, it is $Q = 10 - P$. With probability .5, it is $Q = 16 - P$.

i. What is the efficient tax? In other words, which level of tax minimizes expected dead-weight loss? **Answer** Let $D_1$ represent the state of nature where quantity demanded is $10 - P$ and $D_2$ be the state of nature where quantity demanded is $16 - P$. Note that, in either case, the optimal tax is $4$ per ton. Therefore, even though setting a tax equal to $4$ per ton will result in an uncertain level of pollution (either 6 tons or 12), it will always be efficient. To be really very specific, when the demand is $D_1$, marginal damage and marginal willingto pay intersect at 6 tons which is exactly what setting the tax equal to 4 would achieve when demand is $D_1$. When the demand is $D_2$, MD and MWTP intersect at 12 tons which is exactly what a tax of = $4$ per tone would achieve when the demand is $D_2$.

ii. What is the efficient number of tradable permits to issue (or auction off). That is, again, the number that minimizes expected dead weight loss. **Answer** The first thing to note is that no quota level can ever be perfect here. Setting the quota level at 6 will be right if the demand turns out to be $D_1$, while 12 will be right if it turns out to be $D_2$. You might guess that the best answer is to split the difference and set the quota equal to 9 and you would be right. Here’s why. The expected dead weight loss of any quota level $q$ will be

$$E(DWL) = (.5)(.5)(q - 6)(4 - (10 - q)) + (.5)(.5)(12 - q)(16 - q - 4)$$

The first half of this expression is the contribution to the E(DWL) of setting the quota equal to $q$ when the demand turns out to be $D_1$ - the area below MD and above the $D_1$ between 6 (the efficient level in this case) and $q$ multiplied times the probability of that case. The second half similar represents the contribution to E(DWL) for the other case. Simplifying we get

$$E(DWL) = (.25)(q - 6)^2 + (.25)(12 - q)^2$$

Differentiating w.r.t. $q$ and setting equal to zero, we get
\[ .5(q - 6) = .5(12 - q) \]
\[ \Rightarrow q^* = 9 \]

The figure below shows the DWL associated with setting the quota to 9 under each demand condition. Note that since the two demand conditions are expected with equal probability the marginal DWL \(|MD - MWTP|\) at the solution is equal in both directions.

![Diagram showing DWL associated with setting the quota to 9 under each demand condition.](image)

**Figure ??**

iii. In general, what can you conclude about the efficiency of a quantity approach (tradeable permits) versus a price-based approach (taxes) to regulate pollution? **ANSWER** In general, as we discussed in class, when there is uncertainty over demand, the ideal solution is to have an artificial supply curve for the pollution which mimics to as great a degree as possible the true marginal damage function. In this case, the true marginal damage is constant which is easily and perfectly mimicked by a tax. Conversely, if we faced a situation in which the marginal damages rise steeply (especially over the range of uncertainty in demand), a quantity approach is more likely to have an efficiency advantage over a tax.

iv. (Bonus Point) Whose classic paper titled ‘Prices versus Quantities’ (Review of Eco-
nomic Studies, 1974) established this theoretical result? **ANSWER** Martin Weitzman

**Grading Note** (c) 3 Points for (i); 3 Points for (ii); 4 Points for (iii) 1 Point for (iv). For (i) and (ii) 1 point for right answer, up to 2 more for good explanations. On (iii) 2 points for notion of replicating or mimicing MD curve with intervention policy. 2 more points for explaining when tax or quota does a better job of this.

10. [25 Points] Chad has preferences over consumption outcomes in two states of nature \((x_b, x_g)\) given by the following expected utility function

\[
U(x_b, x_g) = E u(x_b, x_g) = \pi_b u(x_b) + (1 - \pi_b) u(x_g)
\]

where \(\pi_b\) is the probability of state \(b\) and \(u\) is an increasing twice-differentiable concave function. In other words, for all \(x\), \(u'(x) > 0\) and \(u''(x) < 0\). Chad’s endowment is \((\omega_b, \omega_g)\) where \(\omega_b < \omega_g\). Chad has access to an insurance market in which he can purchase up to \(\omega_g - \omega_b\) units of insurance at a price of \(\gamma\) each.\(^1\)

(a) [10 Points] Draw a diagram illustrating two possible budget lines for Chad in \((x_b, x_g)\) space. One budget line should reflect the case where \(\gamma > \pi_b\) The other where \(\gamma = \pi_b\).

**Grading Note** 1 Point for labeling axes. 1 Points for putting endowment point in correct area of the space. 3 Points for having two BLs go through a well-labeled endowment point. 3 points for showing correct relative slope of the two budget lines. 2 more for last two feature both being correct.

(b) [5 Points] Recalling the ‘Full Insurance Principle’ draw two indifference curves - one tangent to each budget line - consistent with Chad’s preferences.

\(^1\)As usual, each unit is a contract that pays $1 only in the even of state \(b\). The premium for each contract, \(\gamma\), is paid before the state of nature is realized and is not refunded in either state.
Grading Note 1 point off for not making clear that FOC is satisfied at 45 degree on $\gamma = \pi_b$ budget line. (tangency, not a kink point solution - even if it is that as well). If indifference curves suggest fully insurance on BOTH budget lines, award 1 point total for this part.
(c) [10 Points] Prove the Full Insurance Principle for this case. Answer Let $k$ be the amount for insurance purchased. Expected Utility as a function of $k$ is

$$Eu(k) = \pi_b u(\omega_b + (1 - \gamma)k) + (1 - \pi_b) u(\omega_g - \gamma k)$$

The optimal choice $k^*$ will satisfy the first order condition $dEu(k^*)/dk = 0.$ which gives us

$$\pi_b(1 - \gamma) u'(\omega_b + (1 - \gamma)k^*) - \gamma(1 - \pi_b) u'(\omega_g - \gamma k^*) = 0$$

In the case of fair insurance ($\gamma = \pi_b$), this becomes

$$\pi_b(1 - \pi_b) u'(\omega_b + (1 - \pi_b)k^*) - \pi_b(1 - \pi_b) u'(\omega_g - \pi_b k^*) = 0$$

which implies that

$$u'(\omega_b + (1 - \pi_b)k^*) = u'(\omega_g - \pi_b k^*)$$

In general, since $u'' < 0$, $u'(x) = u'(y) \Rightarrow x = y$. Therefore, when $\gamma = \pi_b$

$$\omega_b + (1 - \pi_b)k^* = \omega_g - \pi_b k^*$$

$$\Rightarrow k^* = \omega_g - \omega_b$$

**Grading Note** Most scores are either close to zero or close to 10. Proof had to be basically correct to get much credit. Minor errors came with 1 or 2 point deductions.