

HOMEWORK 2 PREFERENCES AND UTILITY

W & L INTERMEDIATE MICROECONOMICS
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- (1) Consider the preferences for bundles of Good 1 and Good 2 depicted below. Assume that preferences are monotonic (more is better).

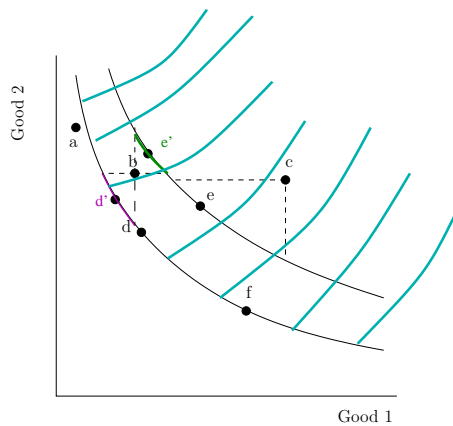


Figure 1.

- (a) Rank the bundles from most preferred to least preferred noting any ties. **ANSWER.** $c \succ e \succ b \succ d \sim f \succ a$. To see why consider first the assertion that $c \succ e$. This claim follows from monotonicity alone: The bundle e has less of both goods and is therefore ranked strictly below bundle c . This is indicated in the augmented figure 1 by showing that e lies below the horizontal line running through c and left of the vertical line running through c . Similarly b has less of both goods than e' and since e and e' lie on the same indifference curve we can conclude that $e \sim e' \succ b$ which by transitivity implies that $e \succ b$. The remainder of the ranking can be justified similarly.
- (b) Shade in area representing all bundles which this consumer likes at least as much as bundle f . **ANSWER.** The entire area indicated by the blue shading in Figure 1 (including the indifference curve on which f lies). In other words, the shaded area represents the upper contour set associated with bundle f .
- (2) Consider the bundles of Good 1 and Good 2 depicted below. The two diagrams show the exact same bundles. Assume that both Jaime on the left and Lucy on the right are rational and have convex monotonic preferences.

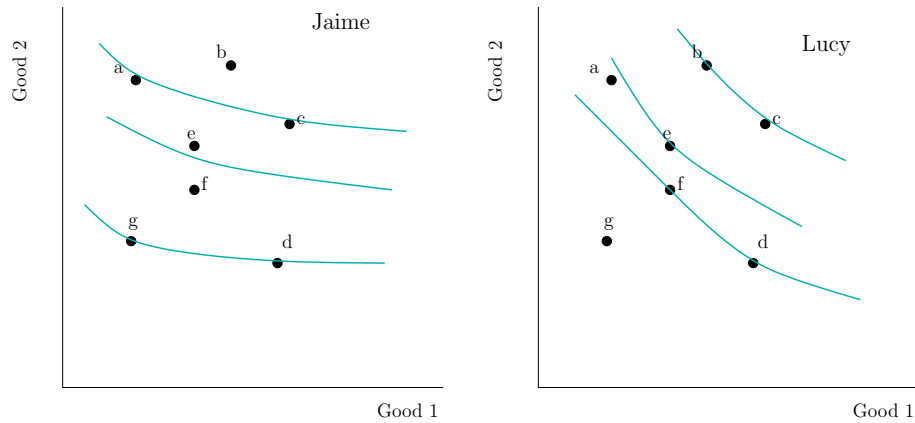


Figure 2.

(a) Draw Jaime's indifference curves so that they reflect the following preference ordering.

$$(1) \quad b \succ a \sim c \succ e \succ f \succ g \sim d$$

ANSWER See figure 2. Be careful that your drawing is consistent with the ranking *and* with the assumptions stated that both individuals have convex and monotonic preferences. In particular I often see answers that show indifference curves violating monotonicity in that they “slope upward” - that is they run in the direction of more of both goods.

(b) Draw Lucy's indifference curves so that they reflect the following preference ordering.

$$(2) \quad c \sim b \succ e \succ a \succ f \sim d \succ g$$

By the way, \sim means “is as good as”; \succ means “is strictly preferred to”. So $x \succ y$ means that x is strictly preferred to y .

ANSWER See figure 2.

(3) *Strict Preferences* (Optional). Define \succ as follows. $x \succ y$ if and only if $x \succsim y$ and $y \not\succeq x$. Read this as x is *strictly* preferred to y . Assume that the commodity space is such that “ties” are possible (i.e. assume there exist indifference sets with more than one bundle). Assume also that \succsim is rational.

(a) Is \succ complete? **ANSWER** No. *Rough Proof.* In order for \succ to be complete, we must have either $x \succ y$ or $y \succ x$. Suppose that for some x and y we have $x \succsim y$ AND $y \succsim x$. (i.e. $x \sim y$ by the standard definition of \sim .) Then by the definition of \succ

$$x \not\succeq y$$

$$(3) \quad y \not\succeq x$$

Therefore \succ is not complete.

(b) Is \succ transitive? **ANSWER** Yes.

Proof. Suppose that $x \succ y$ and $y \succ z$. We need to show that $x \succ z$. By the definition of \succ this gives us the following four results

- (i) $x \succsim y$
- (ii) $y \not\succeq x$
- (iii) $y \succsim z$
- (iv) $z \not\succeq y$

Also using (i), (iii) and the transitivity of \succsim we have

$$(v) \quad x \succsim z$$

Now to complete the proof we need to show that $z \not\succeq x$. To show this, use proof by contraction: Suppose that $z \succsim x$. Then by (iii) and the transitivity of \succsim we would conclude that $y \succsim x$. But this contradicts (ii). Therefore it must be that $z \not\succeq x$. This fact along with (v) and the definition of \succ means that $x \succ z$. QED.

(4) *Condorcet.* Suppose that Allen (A), Betty (B) and Caroline (C) each have rational (complete and transitive) preferences when it comes to choosing between seeing the three movies *Dancing Wolves* (W), *Marching Penguins* (P) and *Crouching Tigers* (T). Allen's preferences are as follows.

$$W \succ_A P \succ_A T$$

Meanwhile Betty's and Caroline's preference are describe as follows.

$$P \succ_B T \succ_B W$$

$$T \succ_C W \succ_C P$$

Defined a preference relation \succsim_m according to pair-wise majority-rule elections. That is define \succsim_m , so that $x \succ_m y$ if x would get at least 2 of the votes when both x and y are put to a vote and $x \sim y$ if x and y get an equal number of votes (which will never happen with three voters). Show that \succsim_m is irrational.

ANSWER. In order to completely describe \succsim_m , we need to run elections between each pair of choices and determine which choice would be the majority-rule winner for from each pair. The results are summarized below.

Election	A's Vote	B's Vote	C's Vote	\succsim_m Result
P vs T	P	P	T	$P \succ_m T$
T vs W	W	T	T	$T \succ_m W$
W vs P	W	P	W	$W \succ_m P$

Looking at the last column we see that \succsim_m clearly violates transitivity and is therefore not rational.

(5) Give an example of preferences, by sketching indifference curves which are ...

(a) convex but *not* monotonic.

ANSWER. See figure 3.

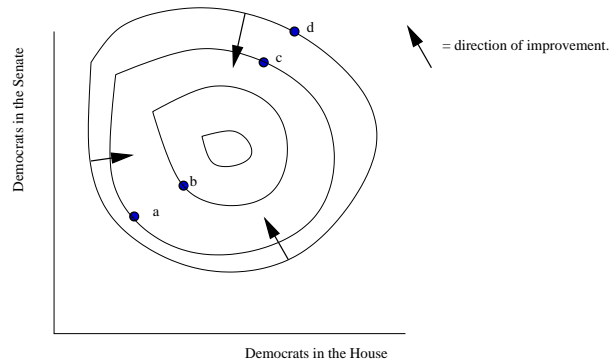


Figure 3. This individual's preference for democrats in the two legislative houses is convex but not monotonic. Note that whenever non-monotonic preferences are drawn the direction of improvement is ambiguous. Therefore it should be noted explicitly. In this case the solid arrows indicate which direction is "up". The violation of monotonicity can be seen directly in the fact that c has fewer democrats in both chambers and yet is preferred to d . This, by itself would not really be a problem since we could just recast the space in terms of republicans and says that more republicans is better for this individual in the neighborhood of c and d . The more substantive violation of monotonicity lies in the fact that in some regions more democrats in both chambers is better (i.e. $b \succ a$), while in other the more democrats in both chambers is worse.

(b) monotonic but *not* convex.

ANSWER. See figure 4.

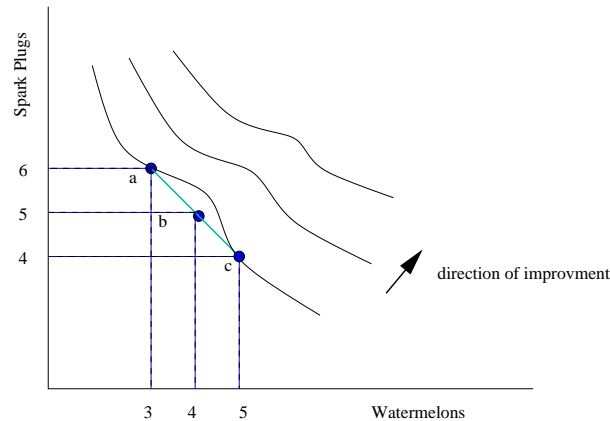


Figure 4. This individual's preferences for spark plugs and watermelon is monotonic but not convex. We can see that convexity is violated by examining bundles a and b and c . While the consumer is indifferent between a and c , she prefers both to b which is the average of the two - average number of watermelons and average number of spark plugs. Hence the upper contour set generated by a (or c) is not convex.

(6) Camilla Dover has preferences over consumption bundles of pizza (Z) and beer (B) represented by the utility function, $u(Z, B) = Z^{\frac{1}{8}} B^{\frac{1}{8}}$. Meanwhile Carson Doogle has preferences represented by $u(Z, B) = Z^2 B^2$.

- (a) Draw a picture of Camilla’s preferences in $Z \times B$ space by sketching a few of her indifference curves
- (b) Draw a picture of Carson’s preferences in $Z \times B$ space by sketching a few of his indifference curves

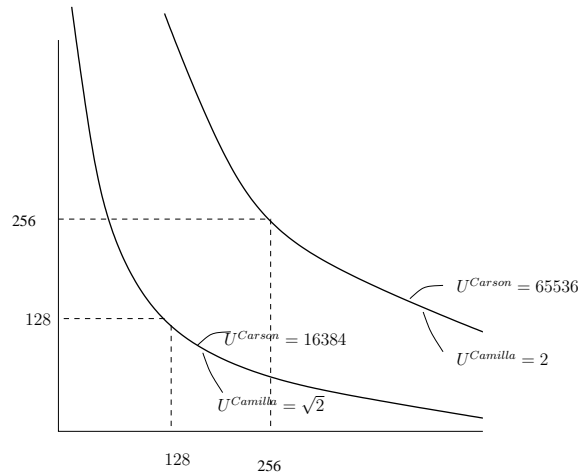


Figure 5. The picture of Camilla and Carson’s indifference curves will look *exactly the same*. The ICs would be labeled with different numbers but those labels would rank the ICs in the same order. In particular, for any consumption bundle Carson’s utility will always be equal to Camilla’s utility raised to the 16th power. Since $f(x) = x^{16}$ is a strictly increasing function, Carson’s utility function is a monotonic transformation of Camilla’s.

- (c) Dr. Ity a senior Vatican official and amateur economist says, “We only have 5 pizza and 3 beers to give to Camilla and Carson. To maximize total welfare we should give all of it to Carson.” Critique Dr. Ity’s statement.**ANSWER.** Since the utility function used to represent Carson’s preferences is just a monotonic transformation of the one used to represent Camilla’s preferences, these two individuals have the exact same preferences for pizza and beer: Any question posed to either consumers about willingness to exchange one commodity for the other from any starting point, would be answered in exactly the same way. This is as far as we can go in using the utility function to compare Camilla and Carson. Any conclusions drawn from the cardinal properties of the utility functions would be an abuse of the theory. So Cardinal Ity’s allocation recommendation will literally maximize total utility but in general the utility total generated by an allocation is not indicative of “welfare” and we cannot say which allocation will maximize societal welfare without making further assumptions about what that means.