(1)  (a) Suppose that your utility over wealth outcomes is given by \( u(c) = \log(c) \).
There is a ten percent chance that tomorrow your house will slide off the side of the hill it is sitting on. Then again, it might not, with probability .9. Your house is worth 300,000 and the rest of your wealth - which would be unaffected by the mudslide - is 100,000. Assume that after tomorrow, if the house didn’t slide off the hill, it never will.

(b) What is the most a risk neutral person would pay for your house? **ANSWER** The expect value of the house is \( .1(0) + .9(300K) = 270K \). Risk neutral people maximize expected wealth as so should be willing to pay an amount up the expected value just calculated.

(c) What is the least you would be willing to accept for your house? **ANSWER** The uninsured home owner currently has expected utility as follows.

\[
Eu(c) = .1 \log 100000 + .9 \log 400000 \\
= 1.151292 + 11.609297 \\
= 12.760589
\]

The certainty equivalent is gotten by solving

\[
u(CE) = Eu(c) \\
\Rightarrow \log(CE) = 12.760589 \\
\Rightarrow e^{\log(CE)} = e^{12.760589} \\
\Rightarrow CE = 348,220
\]

Hence you should be willing to trade you entire portfolio (house and other assets) for $348,000 in certain wealth. Since you have $100,000 in non-house assets, you should be willing to sell you house for as little as $248,220.
(d) If you could buy fair insurance, how much coverage would you purchase? 

**ANSWER.** The fair price is equal to the probability of the bad state, so $0.10 per $1.00 of coverage. If you bought $300,000 at this price, you would pay $30,000. Therefore you end up with the same amount of wealth in both states of nature. ($370,000). You would be fully-insured. WARNING, if the price of insurance is more than the fair price, you will not fully-insure, though you may still buy some insurance.

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**Willingness to Accept for Risky Asset**

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1. Al owns 100 shares in a pumpkin farm, PumpKo. The pumpkin business is very certain. No matter what happens that pumpkin farm will make a profit equal to $100 per share.1 Consequently the market price of PumpKo shares

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1We are assuming only two time periods, so in the next period, PumpKo will pay out $100 per share in dividends, the shareholders will consume those profits and then the world will end.
is exactly $100. On the other hand, shares in Flaxilicious, a flax growing company sell for $80 per share. The Flax business is less certain than pumpkins. With probability .25, flax seed oil will become tremendously popular and Flaxilicious will experience profits of 400 per share. With probability .75, people who eat too much flax seed oil will start to spontaneously combust in large numbers and the company will have zero profits.

(a) Draw a diagram with Al’s consumption consumption possibilities in the two states of nature. Assume that Al cannot sell either stock short. (Is this a reasonable assumption? for both stocks?) Hint: go through these steps...

(i) a good place to start is at the endowment point. How much will Al have in each state if he simply keeps his 100 shares of PumpKo? Plot this point in your diagram.

(ii) Now how many Flax shares can he buy if he sells a PumpKo share? **ANSWER.** Selling one Pumpko share will raise $100, since Falx shares go for $80, he can buy \( \frac{100}{80} \) or 1.25 Flax shares.

(iii) With only 99 shares in PumpKo and the number of Flax shares you determined in the previous step, how much consumption will Al have in each state of the world? **ANSWER.** One less Pumpko share means $100 less in both states of nature, the extra 1.25 Flax shares means $500 more in the good state and 0 more in the bad state. Hence the net change is -$100 in the bad state and +$400 in the good. In other words the MRT is 4. In yet other words, he get $4 additional in bad state consumption for $1 of good state consumption given up. If we allow him to sell Flax shares short, he his budget line is the entire line extending from point (0,50000) to (12500,0) and passing through the endowment point, (10000,10000).

(b) Suppose Al has utility

\[
u(x_G, x_B) = .25 \log x_G + .75 \log x_B
\]

where \( x_G \) refers to Al’s consumption in the good state where Flax profits are $400 per share and \( x_B \) refers to Al’s consumption in the bad state when Flax profits are zero. How many shares of Flaxilicious should Al buy, if any? **ANSWER.** First let \( S_F \) be the number of Flax shares he decides to hold and \( S_K \) be the number of pumpkin shares. From the share prices we know that

\[S_K + .8S_F = 100\]
So if he holds 0 flax shares, then $S_F = 0$ and $S_K = 100$, which represents his position at the endowment point. If $S_F = 125$ then $S_K = 0$ and this would represent the choice of selling completely out of the pumpkin shares. If we assuming that he can sell Flax short, but not Pumpko, it means that he can take any position satifying this equation as long as $S_K \geq 0$\(^2\) Now we want to be able to express $x_B$ and $x_G$ in terms of these share holding.

\[ x_G = 100S_K + 400S_F \quad x_B = 100S_K \]

This is directly from the story. In the Good state, Pumpko pays out $100 per share, while Flax pays out $400 per share. Meanwhile in the Bad state, Pumpko pays out $100 per share, while Flax pays out nothing. So no we can write expected utility (the thing the consumer is trying to maximize) completely in terms of $S_F$

\[ u(x_G, x_B) = .25 \log x_G + .75 \log x_B \]

\[ \Rightarrow u(S_K, S_F) = .25 \log (100S_K + 400S_F) + .75 \log 100S_K \]

\[ \Rightarrow u(S_F) = .25 \log (100(100 - .8S_F) + 400S_F) + .75 \log 100(100 - .8S_F) \]

\[ \Rightarrow u(S_F) = .25 \log (10000 + 320S_F) + .75 \log (10000 - 80S_F) \]

Differentiating with respect to $S_K$ and setting equal to zero, we get a FOC for the optimal $S_F$.

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\(^2\) Technically there should probably be restriction on how negative he can go with the Flax short-selling as well, but let’s ignore that. The action is along the “normal” trading portion of his budget line anyway - where he hold positive amounts of both
\[ \frac{du(S_F^*)}{dS_F} = 0 \]
\Rightarrow \frac{(0.25)320}{10000 + 320S_F^*} - \frac{(0.75)80}{10000 - 80S_F^*} = 0
\Rightarrow 80(10000 - 80S_F^*) = 60(10000 + 320S_F^*)
\Rightarrow 8000 - 64S_F^* = 6000 + 192S_F^*
\Rightarrow 2000 = 256S_F^*
\Rightarrow S_F^* = 7.8125

Another way to approach this is by setting \( MRS = MRT \). We know that \( MRT = 4 \) from above. Using the standard formula for MRS, we have

\[ \frac{\partial u(x_B,x_G)}{\partial x_B} \frac{x_G}{\partial u(x_B,x_G)} = 4 \]
\Rightarrow \frac{0.75}{x_B} = 4
\Rightarrow \frac{3x_G}{x_B} = 4
\Rightarrow x_G = \frac{4x_B}{3}

We need to use the fact that the optimal combination of \((x_B,x_G)\) will lie on the budget line as well. The equation for the budget line is \( x_G = 50000 - 4x_B \).\(^3\) Pluggin this into the equation from the FOC we get

\(^3\)We know from above that the MRT is 4. This is the negative slope of the budget line. And we also know that the endowment point is \((10000,10000)\). Using these facts, you can derive the equation for the line.
This answers the question of how much bad-state consumption our consumer will have, but not how many Flax share he needs to buy. Since 9375 is 625 less than his bad state consumption at the endowment point it must mean that he would have to sell 6.25 shares of Pumpko and therefore buy $6.25 \times 1.25 = 7.8125$ shares of Flax which is exactly what we got doing it the other way.