

Homework 3

Utility Functions, The Consumer's Problem, Demand Curves

Econ 210

1. *Perfect Substitutes*. Suppose that Jack's utility is entirely based on number of hours spent camping (c) and skiing (s). Sketch Jack's indifference curves.

$$u(c, s) = 3c + 2s \tag{1}$$

- (a) What is Jack's MRS of hours spent camping for hours spent skiing?
 - (b) Let $p_c = 1$ be the price to Jack of spending an hour camping and p_s the price per hour of skiing. Solve for p_s^* , the price at which a utility-maximizing Jack might mix his time between the two activities. If $p_s > p_s^*$ what will Jack do?
 - (c) Write down Jack's demand functions for both activities.
 - (d) Let $p_c = 1$ and $m = 90$. Sketch Jack's demand curve for skiing (with the price of skiing on the vertical axis). Re-draw the same curve when $p_c = 4$.
2. *Perfect Complements*. Katherine is only made happy by eating portions of ice cream containing *exactly* four parts chocolate to three parts vanilla. She simply will not eat anything else and will not eat ice-cream in any other ratio. Assume however, that she is willing modifying imperfectly balanced dishes of ice cream by disposal.
- (a) Write down a utility function for Katherine and sketch her indifference curves.
 - (b) If the price of chocolate ice cream is \$3 per pint and price of vanilla is \$2 per pint, what is Katherine's utility-maximizing consumption of chocolate ice-cream when her income is \$9?
 - (c) Write down Katherine's demand functions for both flavors of ice-cream.
 - (d) Draw Katherine's Engel curve for chocolate ice-cream under the prices given above and show how it changes when the price of vanilla decreases to \$1 per pint.
 - (e) Sketch Katherine's demand curve for vanilla ice-cream and show how it changes when the price of chocolate ice-cream increases to \$4 per pint.
3. *Cobb-Douglas*. Consider the utility function $u(x_1, x_2) = x_1^a x_2^b$, $a > 0$, $b > 0$.

- (a) Find the MRS in terms of x_1 , x_2 , a and b .
- (b) Define a new utility function $v(x_1, x_2) = \log(u(x_1, x_2))$. Show that the MRS for v is everywhere the same as the MRS for u .
- (c) Let $a = .4$ and $b = .6$. Write down the demand functions for goods 1 and 2. Sketch the price-expansion path and the demand curve with price on the vertical axis for good 1. Sketch the income expansion path and the Engel curve with income on the vertical axis for good 1.
4. *Quasi-linear Preferences* Consider the utility function $u(x_1, x_2) = x_2 - (x_1 - 5)^2$. Sketch the indifference curves for this utility function.
- (a) Describe in words how this consumer feels about the two goods.
- (b) Write down an expression for the MRS. (i.e. find $\frac{\partial u / \partial x_1}{\partial u / \partial x_2}$) At what rate is this consumer willing to give up good 2 to get another unit of good 1 when $x_1 = 4$. How about when $x_1 = 6$.
- (c) Solve for the demands of both goods.
5. Show as rigorously as possible that in a 2-good world, both goods cannot be inferior. Use any of our usual “nice” assumptions on preferences (rational, monotonic, convex) that you need.
6. (Optional) Recommended Problems from the Bergstrom-Varian Workbook:
- (a) The one from Chapter 6 that begins, “Linus has a demand function with the equation $q = 10 - 2p$ ”.
- (b) The one from Chapter 6 that begins, “Richard and Mary Stout have fallen on hard times...”.
- (c) The one from Chapter 6 that begins, “Douglas has a cousin named Gary Stone...”.
- (d) The one from Chapter 6 that begins, “Under current tax law, certain individuals ...”.