Homework 4
Slutsky Compensation

Econ 210

1. **Linear Demand**. Suppose that Allen's demand for beer measured in pints per month is $b(p_b) = \frac{m}{20} - 5p_b$ and his monthly income m is equal to 1000. Consider a price increase from $p_b = 2$ to $p_b = 3$.

   (a) Find m^c, the amount of income Allen would need to maintain his previous lifestyle [1] under the old lower price.

   (b) Find b^c, the amount of beer Allen would consume if his income were m^c and the price of beer is the new higher price?

   (c) Find the substitution effect, income effect and total effect of this price change.

2. Draw a picture in 2-good space showing a good which is inferior but not Giffen. Be sure to label income and substitution effects.

3. **Labor Supply**. Charles gets utility $u(C, L)$ from consumption, C, measured in dollars and leisure, L, measured in hours. There are 268 hours in a week, but assume that Charles has at most 100 hours per week for leisure since the remainder are required for vital activities such as sleeping, eating and doing chores for his wife. Let w be the wage that Charles receives at the loading dock where he can put in all the hours he wants (up to 100).

   (a) If we always assume that the "price" of consumption, p_c is \$1, what is the price of leisure p_L? [Hint: Consider the MRT; How many dollars of consumption must he give up to get another hour of leisure?]

   (b) Draw a picture of his budget set in $C \times L$ space.

   (c) Suppose that when his wage is \$10 per hour, Charles works 40 hours per week. Plot this point on his budget line. [Hint: (Hours Working) + (Leisure) = 100]

[1] Lifestyle, in the Slutsky sense of previous consumption bundle, not necessarily previous level of welfare, which would constitute Hicksian compensation

(d) Assume that leisure is a normal good. Sketch three points along Charles' *wage expansion path* which show him *decreasing* his consumption of leisure to 50 when his wage goes to \$15 and then increasing it when his wage changes to \$20. Label income and substitution effects. (It may be easier to draw two new pictures to show the decompositions - one for the wage change from 10 to 15, and the other to show the wage change from 15 to 20.) What is going on here? Explain why it is possible for Charles to *increase* his consumption of leisure when its price increases even though it is a normal good?

4. Suppose that if the interest rate is .1, Jack is a borrower. Explain, in terms of income and substitution effects, what happens to his demand for present-day consumption as the interest rate increases.

5. Recall that the present value of an infinite number of annual payments a starting in one year is $\frac{a}{r}$, and a stream of T annual payments starting in one year is $\frac{a}{r} - \frac{\frac{a}{r}}{(1+r)^T}$.

   (a) What is the net present value of a project with costs equal to 25, 25 and 300 in one, two and three years and benefits equal to 100, 100 and 100 in years one, two and three? when the interest rate is .05? when the interest rate is .25? How high does one's opportunity cost of capital have to be in order for this project to look like a good investment?

   (b) Consider a project with costs equal to 600, 100 and 100 in one, two and three years and benefits equal to 300, 300 and 300 in one, two and three years. Describe the net present value as a function of the opporutnity cost of capital, r. How would you describe the difference between this project and the previous one?

   (c) If you have a net worth of 0, but can borrow at r = .1, what is most you would be willing to pay for a piece of farm land that generates \$200 in crops (after planting and harvesting costs) and is assessed \$40 per year in property taxes?

   (d) If you can only afford annual mortgage payments of \$8000, what would the interest rate have to be in order for you to get a 30 year mortgage of \$200,000?

6. **Cobb-Douglas (Optional)**. There is an elegant relationship between the income and substitution effects in CD demands which depends on the weight parameter α which we will explore here. Let $u(x_1, x_2) = \alpha \log x_1 + (1-\alpha) \log x_2$. We will consider a price decrease for good 1 from $p_1 = p_H$ to $p_1 = p_L$ where $p_L < p_H$.

   (a) Write down the demand for good 1 when the prices are p_H and p_L. That is write down $x_1(p_H, m)$ and $x_1(p_L, m)$.

   (b) What is the *total effect* of the price change from p_H to p_L on the demand for good 1?

(c) If income is m, what is m^c – the compensating level of income at which the consumer would be able to just afford the bundle she demanded when the price was p_H *after* the price drops to the new low price, p_L?

(d) $x_1(p_L, m^c)$ is the consumer's demand for good under at the new low price p_L when he has only the compensating level of income m^c to spend. Show that

$$x_1(p_L, m^c) = \frac{\alpha m \left[p_H + \alpha(p_L - p_H)\right]}{p_H p_L}$$

(e) Show that the substitution effect is

$$SE: \quad x_1(p_L, m^c) - x_1(p_H, m) = \alpha(1-\alpha)\theta$$

where $\theta = \frac{m(p_H - p_L)}{p_H p_L}$. Interpret this.

(f) Show that the income effect is

$$IE: \quad x_1(p_L, m) - x_1(p_L, m^c) = \alpha^2\theta$$

where again $\theta = \frac{m(p_H - p_L)}{p_H p_L}$.

(g) Draw a graph with α on the horizontal axis (going from 0 to 1) and change in demand for good 1 on the vertical axis. Plot the *total effect*, from part (b), and the *income effect* together in the same graph. Do *not* plot the substitution effect. Label those two plotted lines. Also label the substitution effect (even though you didn't draw a curve for it). Describe what happens to the income effect as α approaches 1.

Some Hints...

i. The scale along the vertical axis should go from 0 to θ.

ii. Write down the expression for the total effect in terms of θ.