(1) (Optional) The SWS makes sweaters out of labor and capital. The following production function represents their technology

\[ f(x_l, x_k) = 80 \log \left( \min \left\{ \frac{x_l}{2}, x_k \right\} + 1 \right) \]

(a) How many sweaters are made with no inputs?
(b) Using isoquants, draw a picture of this technology in labor × capital space.
(c) Derive the factor demands for labor and capital.

(2) Arlo Corp makes toy wagons out of labor, \( l \), and machines, \( k \). Their output is given by the following production function

\[ f(x_l, x_k) = x_l^a x_k^b \]

where \( x_l \) is the quantity of labor used per day; \( x_k \) is the number of machines they own.

(a) Write down the first order conditions for profit maximization - one for labor and one for capital.
(b) Assume \( a + b < 1 \). Solve for Arlo’s factor demands, given the price of output \( p \) and factor prices \( w_L \) and \( w_K \).
(c) Now assume \( a + b = 1 \). Now RTS is constant. Assume that instead of a constant wage rate, Arlo faces an upward sloping local labor supply curve given by \( w_L(x_L) = \alpha x_L \). Solve for factor demands.

(3) Consider the first-price sealed bid auction in which each player’s true willingness to pay \( v_i \) is an independent draw from \( U(0, 1) \).

(a) In the 2-player game, show that if player 2 follows that strategy \( b_2(v_2) = \frac{v_2}{2} \), then player 1’s best response is to pursue the strategy \( b_1(v_1) = \frac{v_1}{2} \).
(b) In the 3-player game, show that if players 2 and 3 follow \( b_j(v_j) = \frac{2v_j}{3} \), then player 1’s best response is to pursue the strategy \( b_1(v_1) = \frac{2v_1}{3} \).

(4) Consider a game with 2 bidders and 2 objects. The bidders’ values for each object are random draws on \( U(0, 1) \). In other words there are four values:
where $v_{i,j}$ is defined as bidder $i$'s willingness to pay for object $j$. In all cases below, dead-weight loss is the value of possible un-realized gains from trade. For example, if Player 1 ends up with object 1 and $v_{1,1} > v_{2,1}$, then there is no dead weight loss incurred by the allocation of that object since Player 1 values it more than Player 2 does. However, if instead $v_{1,1} < v_{2,1}$, then there is a dead-weight loss of $v_{2,1} - v_{1,1}$ incurred by having Player 1 ending up with object 1.

(a) Calculate the expected dead-weight loss when the objects are allocated among the two individuals by flipping a coin for each object. Each time the coin lands heads up, the object goes to Player 1. On tails, the object goes to Player 2. (note that one player could end up with both objects)

(b) Calculate the the expected dead-weight loss when the objects allocated as follows. Each player submits a preference ranking (assume they tell the truth, so that, for example, if $v_{1,1} > v_{1,2}$, then Player 1 honestly expresses a preference for object 1.) If the rankings are compatible in the sense that the two players prefer different objects, then the allocation is according to the preferences. If the players express the same preference ordering, then we fall back on flipping coins to determine the allocation as in the last part.