

ECON 210 - MICROECONOMIC THEORY
HOMEWORK 6
PROFIT

PROFESSOR JOSEPH GUSE

- (1) The SWS makes sweaters out of labor and capital. The following production function represents their technology

$$f(x_l, x_k) = 80 \log \left(\min \left\{ \frac{x_l}{2}, x_k \right\} + 1 \right)$$

- (a) How many sweaters are made with no inputs?
(b) Using isoquants, draw a picture of this technology in labor \times capital space.
(c) Derive the factor demands for labor and capital.
- (2) Arlo Corp makes toy wagons out of labor, l , and machines, k . Their output is given by the following production function

$$f(x_l, x_k) = x_l^a x_k^b$$

where x_l is the quantity of labor used per day; x_k is the number of machines they own.

- (a) Write down the first order conditions for profit maximization - one for labor and one for capital.
(b) Assume $a + b < 1$. Solve for Arlo's factor demands, given the price of output p and factor prices w_L and w_K .
(c) Now assume $a + b = 1$. Now RTS is constant. Assume that instead of a constant wage rate, Arlo faces an upward sloping local labor supply curve given by $w_L(x_L) = \alpha x_L$. Solve for factor demands.
- (3) Suppose that the Rigel Corporation makes electricity (Y) using coal (C) and labor (L) and land (D) according to the following production function

$$Y = 200C^{\frac{2}{5}}L^{\frac{1}{5}}D^{\frac{2}{5}}$$

- (a) What are the returns to scale of Rigel's electricity production? Show your work.

- (b) Suppose that Rigel's land holdings are fixed in the medium-run. What are the returns to scaling up just the coal and labor inputs?
 - (c) What is the marginal product of labor? of coal?
 - (d) Show that the marginal rate of technical substitution between labor and land is decreasing.
 - (e) Write down an expression for Rigel's profit as a function of prices and input levels. Use p_Y for the price of electricity, w_C for the price of coal, w_L for the price of labor, and w_D for the price of land.
 - (f) Write down the first order conditions for profit maximization
 - (g) Suppose that in the short run labor (L) and land (D) are fixed at levels, \bar{L} and \bar{D} . Draw a picture in $Y \times C$ space showing the production function, iso profit lines and the firm's profit maximizing choice of coal. How does the price of electricity affect the optimal choice of coal?
- (4) Julie makes cookies (Y) using labor (L), sugar (S) and corn-syrup (R) according to the following production function.

$$Y = 100L^{\frac{3}{7}}(2S + R)^{\frac{3}{7}}$$

- (a) What are Julie's returns to scale?
- (b) What is MRtS between sugar (S) and corn syrup (R)? Interpret your answer in words.
- (c) What is the MRtS between labor (L) and sugar (S)? Interpret your answer in words.
- (d) (Optional) Suppose that the price of sugar, $w_S = 3$, the price of labor $w_L = 10$ and the price of corn syrup $w_R = 1$. How many cookies should Julie make as a function of the price of cookies p_Y ?