LECTURE NOTE 1
BUDGET SETS

W & L INTERMEDIATE MICROECONOMICS
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The classic model of consumer decision making runs something as follows: *Consumers make themselves as well off as they are able.* This statement can be broken down into two parts. First there is the notion that consumers always want to make themselves better off. Second is the admission that they are constrained in this pursuit - "as they are able". We will begin by addressing each of these notions separately. In the sections titled Preferences and Utility we will introduce ways to model welfare and happiness. In this note, Budget Sets, we will introduce how economists model the resource limitations consumers face. Finally in the section titled The Consumer's Problem, we will bring these two notions together in a formal model of consumer decision making.

1. Consumption Bundles, the universe of consumer outcomes.

Roughly speaking a consumption bundle is an array of quantities of different goods and services. For example, you can think of the collection of 2 bananas and 3 haircuts as a consumption bundle, while 4 bananas, 1 haircut and year's worth of cable television as another bundle. However, to be completely accurate we should think of a consumption bundle as not simply a collection of purchasable goods and service, but as those things along with everything about them that is relevant to the consumer's experience. So that 4 bananas and 1 haircuts delivered next Tuesday while it raining out and two days after a job interview is a different consumption bundle than the same number of bananas and haircuts consumed at a different time and place and under different circumstances.

Let I stand for the set of all conceivable goods and services *at the time and in the place the consumer is making her choice.* Think $I = \{$footballs, bananas, cellphone minutes, beer, airline tickets, gasoline, coffee, etc $\}$. A SINGLE consumption bundle is a vector of quantities - one for each possible good or service.

2. Budget Sets

In this section we will talk about how to model what a consumer *is able* to do. We will begin in a simplified environment where consumers have a fixed amount of income and quantities of different commodities or services are available at fixed prices. Let m stand for the amount of income a consumer has, say in dollars. Let p_i stand for the price of good i in dollars per unit. Let x_i stand for quantity purchased of good i.

2.1. Definitions.

The consumers *budget constraint* is given by the following inequality

$$\sum_{i \in I} p_i x_i \leq m \tag{2.1}$$

or if you prefer without the summation sign

$$p_{footballs} x_{footballs} + p_{cell-minutes} x_{cell-minutes} + p_{beer} x_{beer} + \ldots \leq m$$

The left-hand side (lhs) of the budget constraint adds up how much the consumer is spending in dollars. Each term $p_i x_i$ - the price of good i multiplied by the quantity of good i - tells us how much the consumer is spending on good i. The right-hand side (rhs) of the budget constraint tell us how many dollars the consumer has available to spend. Therefore the budget constraint says nothing more than the consumer cannot spend more dollar than she has.

Let $\mathbf{x} = \{x_i\}_{i \in I}$ be a *consumption bundle.* $\mathbf{x}$ represents an entire vector of the consumer's purchasing decisions. That is, $\mathbf{x}$ is a list of quantities - one for footballs, one for cell-minutes, etc. A particular bundle, $\mathbf{x}$, may or may not affordable. That is it may or may not satisfy the budget constraint, inequality 2.1. A consumption bundle which does satisfy the budget constraint is said to be in the consumer's *budget set*, alternatively referred to as the *constraint set*, *choice set* or *opportunity set* (Perloff). Formally, the budget set is given by

$$\left\{ \mathbf{x} : \sum_{i \in I} p_i x_i \leq m, \ \ x_i \geq 0 \ \ \forall i \right\}$$

Notice that besides satisfying the budget constraint, we will also typically require that a consumption bundle only have non-negative quantities. (You can't eat -2 bananas.)

It is important to remember what consists of what. A consumption bundle is a list quantities. It is a vector. A budget set is comprised of all the affordable consumption bundles. It is a set of vectors.

We will also talk about the *budget line*. This is the set of consumption bundles which are *just* affordable.

$$\left\{ \mathbf{x} : \sum_{i \in I} p_i x_i = m, \ \ x_i \geq 0 \ \ \forall i \right\}$$

Notice that the inequality in the budget constraint has been replaced by an equality. Notice also that this makes the budget line a subset of the budget set. Why?

2.2. **A 2-Good World.** We often assume that there are only two goods, good 1 and good 2. Let's re-write our definitions under this assumption.

Budget Constraint:

$$p_1 x_1 + p_2 x_2 \leq m$$

Budget Set:

$$\{(x_1, x_2) : p_1 x_1 + p_2 x_2 \leq m, \ \ x_1 \geq 0, \ \ x_2 \geq 0\}$$

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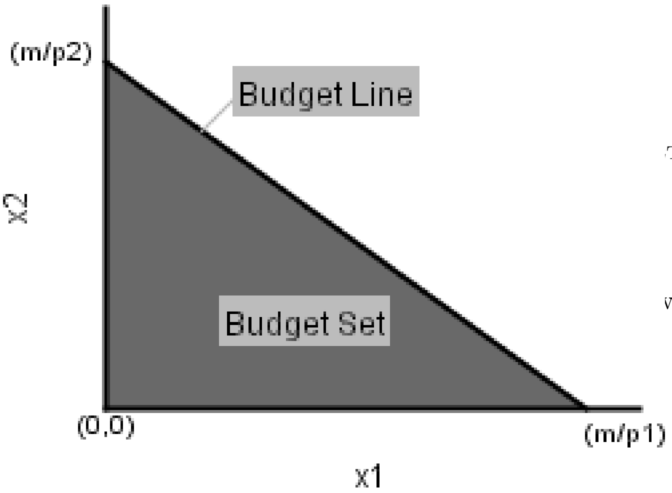

There are several things to note about this picture.

(1) *The intercepts.* The intercepts represent the consumption bundle one would have if the consumer dedicated her entire income to one good or the other. Example, if the consumer had $100 to spend and spent it all on flower pots at $2.50 per glass, then one intercept would be the consumption bundle consisting of 40 flower pots and 0 of all other goods. In general the intercept for good i is given by $\frac{m}{p_i}$.

(2) *The Budget Line is the set of all linear combinations of the intercepts.* In the picture, we see this as the straight line connecting the two intercepts. Formally we can say that a consumption bundle $\mathbf{x} = (x_1, x_2)$ is on the budget line if for some $\alpha \in [0,1]$ we have $x_1 = \alpha\frac{m}{p_1}$ and $x_2 = (1-\alpha)\frac{m}{p_2}$. **Prove this.**

(3) *Marginal Rate of Transformation (MRT).* The negative slope of the budget line is called the *MRT*. It is the rate at which a consumer *can* exchange one good for the other *along the budget line.* Note that the inverse a MRT is also an MRT. For example, $\frac{p_1}{p_2}$ is how many additional units of good 2 one can have by giving up a single unit of good 1, while $\frac{p_2}{p_1}$ is how many additional units of good 1 one can have by giving up a single unit of good 2. **Show that slope of the budget line is** $-\frac{p_1}{p_2}$.

(4) *Price Changes Swing the Budget Line.* To see this keep your eyes on the intercepts. For example, the price of of good 1 increases, the intercept on the good 2 axis remains fixed, while the intercept on the good 1 axis move closer to the origin. **Describe this change in terms of the MRT and the slope**.

(5) *Income Changes Shift the Budget Line.*

3. Examples

Anytime a decision maker faces a trade-off, a Budget Set or similar notion will be relevant. These show up in many contexts. Here are some examples.

3.1. **A Standard Two-Good Budget Set.** Paul has $150 to spend on potatoes and chicken. The price of chicken is $1.50 per pound. The price of potatoes is $.75 per pound.

3.2. **Two Period Consumption.** Alan expects to live for two periods. He will earn $1000 in the first period and $500 in the second period. He can borrow and save at interest rate r. (Here 'good 1' is period 1 consumption and 'good 2' is period 2 consumption.)

3.3. **Leisure and Consumption.** Candice consumes leisure time and other goods which can be purchased with cash. She has no assets other than her ability to work up to 80 hours per week. She earns \$20 per hour.

3.4. **States of Nature / Risk.** Elisa is uncertain about the future, but knows there are exactly two possible states of nature - one where it will be snowy and cold and the other where it will be hot and dry. Her consumption in the future will depend on where she invests her money today. Jan's XC Ski Company will be worth \$2 per share if it turns out to be cold and snowy and \$0 per share otherwise. Wilma's Desalination Industries will be worth \$0 per share if it turns out to be cold and snowy and \$1.50 per share otherwise. Both companies are currently selling for \$1 per share. Elisa has \$1000 to invest.

3.5. **Production Possibilities.** Fred can make two loaves of bread per hour or catch 3 pounds of fish per hour. He can commit up to 10 hours each day either backing or fishing