B. Numerical Results from 20,000 5-Player Games.

B.1 Support Profiles by Ideal Point.

If one solves the equilibria for enough games, a pattern begins to emerge. I already stated in Section 4.4 some of the more obvious patterns that emerged from the analysis performed on 20,000 5-player games. One of those observation was that there is a set of four support profiles (out of over 9000 consistent support profiles) one of which provides the basis of an equilibrium strategy profile in every game analyzed. In Observation 2, I noted that the boundaries between regions of support profiles in the space of games were not linear. However a precise description of the shapes of these regions is difficult to write down. Nevertheless there is certain regularity in the shapes which is undeniable. This appendix is partial remedy to that problem. In this section, I present seven snapshots of these shapes. Each picture represents all the games analyzed at a particular setting of player 1’s ideal point.\(^1\) Each cell in the grid of a picture is associated with a particular combination of player 2 and player 3’s ideal points. Hence every colored cell on each page represents a unique configuration of ideal points. The color of each cell indicates which of the four support profiles - 3L1R, 3L2R, 2L3R, 1L3R or some combination of them - formed the basis of the stationary equilibria found by the numerical search algorithm for that game. From time to time the search algorithm failed to settle on an equilibrium for a given game with the tolerance parameters set for it. The cells representing these games are colored bright orange. Otherwise, the support profile(s) forming the basis of equilibria for any game, can be determined by matching the color of the cell to the key provided in Figure 10.

For this appendix I have chosen to include only a portion of the results. Those who are interested in exploring these results further including data on the reduced strategy profile arrays and expected utility vector for each game are welcome to peruse them further at my website

http://home.wlu.edu/~gusej/coalitions/index.html

\(^1\)Recall that for each of the 20,000 games analysed, player 0’s ideal point was always set to 0 and player 4’s ideal point was always set to 1. That leaves 3 degrees of freedom. The pictures are sorted by the value of player 1’s ideal point.
Figure 10. Color Key. Each little square in the map on the following pages contain a color which indicates which support profiles were found to represent equilibrium strategy profiles in that game. This is a key to those colors. As explained in the main body of the paper, $3L1R$ is the support profile where coalitions $\{0, 1, 2\}$, $\{0, 1, 3\}$, $\{0, 1, 4\}$ and $\{2, 3, 4\}$ are in the support of all the players who are members of them. $3L2R$ is the support profile where coalitions $\{0, 1, 2\}$, $\{0, 1, 3\}$, $\{0, 1, 4\}$, $\{1, 3, 4\}$ and $\{2, 3, 4\}$ are in the support of all the players who are members of them. $2L3R$ is the support profile where coalitions $\{0, 1, 2\}$, $\{0, 1, 3\}$, $\{0, 3, 4\}$, $\{1, 3, 4\}$ and $\{2, 3, 4\}$ are in the support of all the players who are members of them. $1L3R$ is the support profile where coalitions $\{0, 1, 2\}$, $\{0, 3, 4\}$, $\{1, 3, 4\}$ and $\{2, 3, 4\}$ are in the support of all the players who are members of them.
In this picture we see which of the four support profiles form the basis of equilibria for games with ideal points \( \{0, \frac{1}{64}, x_2, x_3, 1.0\} \) where \( \frac{1}{64} \leq x_2 \leq x_3 \leq 1.0 \). Note that support profile 3R1L appears to dominate for low values of \( x_2 \) and high values of \( x_3 \) in the range just specified. Note also that neighborhood around the borders between the regions where support profile dominates tends to exhibit equilibria based on more than one of the four support profiles.
Figure 12. Here we see which of the four support profiles form the basis of equilibria for games with ideal points \( \{0, \frac{8}{64}, x_2, x_3, 1.0\} \) where \( \frac{8}{64} \leq x_2 \leq x_3 \leq 1.0 \).
Figure 13. Here we see which of the four support profiles form the basis of equilibria for games with ideal points \( \{0, \frac{12}{64}, x_2, x_3, 1.0\} \) where \( \frac{12}{64} \leq x_2 \leq x_3 \leq 1.0 \)
Figure 14. Here we see which of the four support profiles form the basis of equilibria for games with ideal points \( \{0, \frac{16}{64}, x_2, x_3, 1.0\} \) where \( \frac{16}{64} \leq x_2 \leq x_3 \leq 1.0 \)
Figure 15. Here we see which of the four support profiles form the basis of equilibria for games with ideal points \( \{ 0, \frac{20}{64}, x_2, x_3, 1.0 \} \) where \( \frac{20}{64} \leq x_2 \leq x_3 \leq 1.0 \).
Figure 16. Here we see which of the four support profiles form the basis of equilibria for games with ideal points \( \{0, \frac{24}{64}, x_2, x_3, 1.0\} \) where \( \frac{24}{64} \leq x_2 \leq x_3 \leq 1.0 \).
Figure 17. Here we see which of the four support profiles form the basis of equilibria for games with ideal points \( \{0, \frac{32}{64}, x_2, x_3, 1.0\} \) where \( \frac{32}{64} \leq x_2 \leq x_3 \leq 1.0 \)

B.2 Examples of Games with Multiple Equilibrium Support Profile Representations.

Observation 3 says that for a given game, all equilibria have identical expected utility vectors. This observation is, in part, analytically proved in Lemma 29 which says that if a strategy profile solves the system of expected utility equations, then any strategy which has the same total weight on each coalition (when summed across players) also solves the system and moreover generates the same expected utility vector. With a little work (and perhaps a
little faith?)\(^2\), it can be shown that this means that all the equilibria represented by the same support profile, must have the same vector of expected utilities. But what about equilibria represented by different support profiles? It does not seem that we can use Lemma 29 to justify the claim that such equilibria should share the same vector of expected utilities in their solution. After all, they certainly do *not* share the same vector of coalition weights in their solution. (They cannot since they are based on different support profiles.) Therefore, it seems remarkable that the numerical in all 20,000 or so game I analyzed appear to bear this out in every case where there were equilibria represented by distinct support profiles. I am confident that there is an analytical proof that will establish this, but I have not yet been able to write it down. In the meantime, here is example of a game where the support profiles 2L3R and 3L2R both represent equilibria. Figures 21 and 20 both show equilibrium solutions to the game with ideal points \(\{0.0000, 0.1250, 0.4531, 0.7031, 1.0000\}\). Note that the coalition weights are radically different, while the expected utility vectors are virtually identical. It also interesting to note the distribution of expected utilities itself. The median player (# 2) gets the most utility on average from playing the game? and how the more extreme you are the less expected utility you get? This is true for all the games analyzed, and it is something the infinite horizon game has in common with the Two-Period Game. This is despite the fact that player 2 is included in fewer coalitions. In other words, the inclusion rate is proportional to expected ideological loss. The more extreme you are the more you are included, though you are still not included enough to entirely make up for the ideological pain you experience on average due to your extreme preferences. Conversely the more moderate you are the less you are included, but the median player - usually the most moderate, is still included frequently enough that she enjoys the most expected utility. It appears that all the same which were at work in the two-period game carry through to the infinite horizon game!

\(^2\)Recall that the number of unknowns and te number of non-redundant equations in the the systems are equal. Therefore if we believe that the solution for any given system is unique, then this is true. The catch is that the combined system is not linear, though there are no higher terms, so I think this is OK.
Figure 20. Support profile 3L2R is the basis for the equilibria represented by this vector of coalition weights.
Figure 21. Support profile 2L3R is the basis for the equilibria represented by this vector of coalition weights.