ELECTING LEGISLATIVE BARGAINERS

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Abstract. When the rules of legislative bargaining involve majority passage, it is not clear how the election of a new member to the legislative body will influence outcomes. For citizens who care about legislative outcomes, this can make the task of voting less than straightforward. In this paper I provide some preliminary characterization of voter incentives in a game where voters elect representatives who go on to play a bargaining game. The outcome of the bargaining game is an allocation of funds to the districts as well as a public policy decision. Both voters and representatives care about the final wealth allocation outcome and there is heterogeneity of preferences in the voting districts for the public policy. I ask how sophisticated voters should vote for members who go on to bargain in the legislature. For example, should a voter with a given ideal point for the public policy outcome try to elect a representative with identical preferences? Preliminary evidence suggests that voters should elect themselves only in rare circumstances. In general the answer depends on the make-up of the remaining representatives in the legislature.

1. Introduction

It is commonly recognized that citizens voting in an election to choose legislative representatives might be “sincere” or “sophisticated”. In models where candidates and voters have ideal points in some compact policy space, “sincere” voting is simply choosing the candidate whose ideal point lies the closest to the voter’s according to some distance measure. In other words, sincere voters do their best to vote for candidates who share their views. Sophisticated voters, on the other hand, will recognize that the best way to achieve a legislative outcome may be to cast one’s vote for a candidate whose preferences are different from one’s own. There are two sources of explanations for optimally choosing a representative different from oneself.

The first source, which is well-studied and with which this paper is not concerned, is the election itself. For example, if there are three candidates, one whom I like, one whom I hate, and one whom I can tolerate, I may vote for the tolerable candidate in favor of the likable one if I believe that the race will come down to the tolerable and hated one. Voting sincerely in such an instance is commonly described as “wasting one’s vote”. Hence non-vote-wasting behaviour can be optimal depending on the voter’s beliefs about fellow voters in the election. In the model at hand, however, I dispense with this issue by having just one voter per legislative district (and hence, one voter per election) which causes this particular brand of difference between sincere and sophisticated voting to disappear.
A second category of reasons to explain a divergence between sincere and sophisticated voting comes out of the legislative process that follows the election. The overarching question in this paper will be, how should sophisticated voters vote when they don’t have to worry about other voters, but they do have to worry about the other legislators? Sophisticated voters should recognize that once elected, a representative’s preferences do not act directly on final legislative outcomes. They act through some sort of bargaining process. This suggests a variety of trade-offs facing sophisticated voters in their quest for the optimal representative. The most obvious trade-off is that between the power one’s representative wields in the bargaining process and the position she advocates. For example, in a legislative environment where the majority rules like the one modeled here, a voter might prefer a candidate representative who can forcefully advocate from within the majority coalition for a position close to one’s own to a candidate who faithfully advocates exactly one’s position but who is less likely to be included in the majority coalition. Less obvious perhaps, the choice of legislative representative may have second-order effects on the bargaining process which suggest subtle but potentially important trade-offs. These have to do, not with the power of one’s own representative, but how the presence of one’s own representative affects the actions of the other legislators. Consider, for example, two candidates running for the same seat, both of whom would be excluded from the majority if they were elected. Supposing that all minority members have the same low level of power when it comes to actually setting policy, shouldn’t a voter be indifferent between them? The answer is no, not if their elections would result in the formation of different majorities. Having one’s representative excluded from a majority which passes agreeable public policy is better than your representative being excluded from a majority which passes disagreeable public policy. The mere presence of a representative, even one which is excluded from the majority in equilibrium, can alter the bargaining power of other legislators as they decide how to form coalitions because it changes the set of coalitions which may be threatened. Truly sophisticated voters should bear this in mind.\footnote{Note that the opportunity to have one’s representative relegated to a powerless minority in more than one way requires that the legislature size be at least 5, which is the size of the legislatures I analyze in the results section. This is the first attempt I know of to model legislative elections in which the legislatures have size greater than 3.}

The model put forward in this paper will attempt to explore the effect of these incentives. The model is a two-stage one-shot game. In the first stage, voters select the ideal point of their representatives. In the second stage, those representatives bargain over a public policy and a pot of money. A distinguishing feature of this model is that voters are allowed to care about both dimensions of the legislative outcomes, both the public policy outcome and the private benefit distribution from the pot. The bargaining game itself is extensively analyzed in Guse (2005) where I describe the equilibrium attributes of two versions - a finite horizon version and an infinite horizon version. For reasons explained below I assume that the bargaining game takes place according to the
rules of the infinite horizon version. I also assume throughout that there are 5 districts represented in the legislature. Analysis is preliminary. In this paper I will report only on my analysis up to characterization of a voter best response function. Charactering the equilibria of the complete two-stage game is left to future work. It is challenging because equilibria in the legislative bargaining subgame itself must be calculated numerically making comparative statics difficult.

The paper proceeds as follows. In section 2, I review two earlier efforts in the literature to analyze integrated legislative elections - models in which voters elect legislators who then play a bargaining game whose outcomes those voters care about. In section 3, I introduce the setup. In section 4, I will discuss the highlights from the numerical analysis of three different legislative settings. The focus here is on how the optimal choice of legislative representative for a voter from one district changes as a function of the voter’s own ideal point for the public policy, the weight the voter puts on public policy (as opposed to the money distribution outcome), and the composition of the other legislators with whom the voter’s representative will have to bargain. Results are preliminary, but largely verify intuition. Voter’s who care more about the public policy than the cash-distribution outcome will tend to optimize in the direction of their own ideal point. Voters who care only about the cash-distribution question should vote for moderates who tend to be included in the majority more often and tend to have more bargaining power when they are.

2. Literature Review

Theories of elections and theories of legislative bargaining each have fairly well-established literatures. However very little has been published to date on integrated models of legislative elections. To my knowledge, there have been only two significant works which have attempted to analyze a multi-stage game in which voters elect representatives who then go on to bargain in a legislature. One is Austen-Smith and Banks (1988, 2005). From here on, I will refer to this model as ASB which was first introduced by those authors in 1988 and then refined for inclusion in a compilation text in 2005. The other is Baron and Diermeier (2001) which I will refer to as BD. These two models are nearly identical. I will consider how their intersection contrasts with the model here.

First they both investigate proportional representation and take parties, not individual legislators to be the fundamental unit of decision-making. That is to say they look at legislatures whose representatives all come, de facto, from the same district. Hence $n$ voters in these models each vote for one party whose platform (ideal point in the policy space) is announced prior to the election. In the most interesting case, all three parties gain representation in the legislature and none with a majority on their own. Then a formateur, analogous to the proposer in this paper, is selected...
according to some mechanism\textsuperscript{2} and must form a majority coalition by choosing to partner with one of the other two parties. By contrast, the model in this paper, has representatives and voters coming from meaningful districts with each district’s winning individual representative getting exactly one vote in the legislature.

The second big difference lies in how voter preferences are specified. In all three models of legislative bargaining (ASB, BD and the one in this paper), legislators care both about some public policy outcome as well as how some pot of private surplus is distributed. However, in both ASB and BD, the domain of voter preferences is limited to the public policy outcome and not the private surplus distribution. By contrast, the model in this paper allows the voters to care about the private surplus distribution to a greater or lesser extent. In fact, the extent to which they do care is captured in a parameter (\(\alpha\)) which gets a lot of attention in my results. The difference between the pot of private surplus in ASB and BD and the pot of private surplus envisioned in this paper goes back to the idea that the model in this paper attempts to capture multi-district elections while ASB and BD effectively analyze a single district election. In their models, the pot of private benefits are thought of as “spoils” of office holding. The typical example they both use are the number of cabinet positions given to members of each party. Thought of in this way, it makes sense that voters would not care \emph{per se} about which party occupies which cabinet posts. In this paper, I prefer to think of the pot of benefits distributed between members as “pork barrel” projects, that is, projects whose benefits accrue primarily to the citizens of one district or another. When thought of in this way, it makes more sense to have both the legislators \textit{and} the district-specific citizens care about the distribution.\textsuperscript{3}

Third, both ASB and BD assume just 3 parties and since parties are the unit decision-makers, the legislatures in both models are effectively size 3. By contrast, the model in this paper will have 5 players in the legislative bargaining stage which allows, for example, for 10 possible minimum majority coalitions instead of 3.\textsuperscript{4} As mentioned earlier having more than 3 minimum majority coalitions allows for the possibility that representatives excluded from the majority can be excluded in more than one way.

In short, both of these integrated models of models of proportional representative legislative elections, ASB and BD, can be thought of a good approximation of many European parliaments,

\textsuperscript{2}Austen-Smith and Banks give the party with the greatest number of votes after the election the first chance to form a coalition, while Baron and Diermeier have the formateur selected at random with weights proportional to their votes shares from the election.

\textsuperscript{3}There is ample anecdotal evidence that members of the U.S. House of Representatives are intensively aware of these preference in their local electorates. Perusing the websites of individual house members, it is easy to find examples, in which the member brags about how many federal dollars they have “brought home” to their district.

\textsuperscript{4}One key result from the analysis of the legislative stage, is that only the 6 “polar” of these 10 is ever formed in equilibrium.
especially if you believe that voters in those elections care only about how well the parties do and not about their own representative *per se*. By contrast the model studied in this paper is perhaps better thought of as elections for seats in a legislature where district origin matters, perhaps the U.S. House of Representatives, or more accurately a city council whose members represents neighborhoods and which decides matters by simple majority.

What did they find? In the words of Baron and Diermeier, ASB is “difficult to analyze and has an abundance of qualitatively different equilibria”. While I would agree with this characterization, I would be suspicious of any model of such a complex game which is easy to analyze and has an unique equilibrium. In ASB, there are 3 parties whose ideal points lie in a one-dimensional policy space. They label these parties *l*, *m* and *r* (left, middle, right). Each party is allowed to choose their ideal point (“platform”) and announce it before the election. *n* voters cast their votes for one of the three parties. In the legislative stage, the party with the highest number of votes is given the first crack at trying to form a majority government (unless they received a majority of votes in the election stage in which case the game is over). They announce a public policy and a distribution of “spoils”. If at least one of the other parties approves (assuming that any two parties of the three is sufficient to make a majority as measured by their vote shares), then the game ends with the formateur’s proposal going into action. If the highest vote-getter fails as formateur, the second-highest vote getter takes a shot. If that party fails, the party who received the fewest votes gets a shot. Finally if that party fails to form a government, some status quo is implemented. The legislative bargaining game is solved by simple backward induction. In that subgame, ASB identify equilibria which are essentially in the same spirit as most of the legislative bargaining model including Guse (2005): The “strong” tend to form majorities with the “weak” and exclude the middle. In the context of the ASB model, this means that the party with the most votes coming out of the election will, as formateur, form a coalition with the party who received the fewest votes. The lion’s share of the spoils go to the formateur with the smallest party getting just enough to agree to the coalition. The public policy is typically the average of the two coalition partners’ ideal points\(^5\). The degree to which this tendency determines the actual outcome depends on the relative vote shares and they distinguish between 7 cases, which I will not go into here. Their main result in the broader election game is that voters who recognize the dynamics of the bargaining game will often have incentive to vote for their 2nd most preferred party platform instead of their first. They also find that voters never vote for their least preferred party. The two coalitions which arise in equilibrium according to ASB are \{l, m\} and \{m, r\}. Furthermore, they find that the middle party announces a platform in equilibrium equal to the ideal point of the median voter and that

\(^5\)Again, Guse (2005) finds similar results
the left and right parties announce platforms significantly deviated from the median but not at the extremes.

The greatest shortcoming of the ASB model, perhaps, is the assumption that the formateur in the legislative bargaining subgame is selected deterministically based on the election results. The model in Baron and Diermeier (BD) attempts to remedy this by having the formateur selected at random with weights proportionate to the vote-shares from the election stage. The other key difference between ASB and BD, is that BD considers a 2-dimensional public policy space. The authors find many interesting results in the legislative subgame involving - among other things - the opportunity for consensus governments to form. (i.e. grand coalitions). This of course has an effect on the incentives voters face in the preceding election stage. Unfortunately their analysis of the legislative bargaining stage is suspect. In Guse (2006b), I show that the payoff function the authors write down used to decide when the formateur should select one coalition or another depends critically on their assumption that formateurs may transfer negative surplus amounts to coalition members. I know of no other divide-the-dollar style bargaining models in which the proposers can do this. It is not surprising that with the sign constraint is lifted from proposals, that proposers will be able to propose efficient all-inclusive outcomes.

3. Setup

There are \( n \) districts each with a single voter. In the first stage of the game, the voters in each district choose a representative. In the second stage, the representatives play a bargaining game. In that bargaining game, two things are determined - an allocation across districts of \( n \) dollars, and some public policy choice on \([0, 1]\). The qualitative difference between the two components of the legislative outcomes is key. The public policy choice is literally public in the sense of non-rivalry and non-excludability. It can be thought of as a decision which has little or negligible impact on private wealth levels but is still something that both legislators and their constituents care about. For example, it seems reasonable to think that whether abortion rights are made more or less restrictive will not have any obvious significant implication for the relative distribution of wealth across districts, but it may be something for which constituents in various districts have very strong and perhaps differing preferences. Hence both the voters’ and the representatives’ utility functions depend on the outcome of the bargaining session as follows.

\[
\begin{align*}
    u_{r(i)} &= y_i - (x - x_{r(i)})^2 \\
    u_{v(i)} &= (1 - \alpha)y_i - \alpha(x - x_{v(i)})^2
\end{align*}
\]

Representative Utility

Voter Utility

where \( y_i \) is the share of the \( N \) in private wealth going to district \( i \) in the final outcome, \( x_{r(i)} \) is the representative’s ideal point for the public policy decision, \( x_{v(i)} \) is the voter’s ideal point for the
public policy, and $x$ is the final public public outcome. The voter here is meant to be representative, perhaps the voter with the median ideal point for the public choice in his or her district.\footnote{One potential problem with this interpretation: There is no guarantee that the preferences over candidates induced by the legislative bargaining game will be single-peaked for all voters in this model. It does appear to be possible to construct voters whose preferences are \textit{not} single peaked. However, in most cases it is difficult to construct such a voter. Also since I am limiting the scope of this paper to the characterization of voter preferences and not attempting as yet to characterize the election equilibrium, I will leave this issue to future work.}

The central question is, how should the voter behave? Should the voter simply elect himself? After all, who better to represent one's will in the legislature than someone with identical preferences? The problem with this conjecture is that one cannot be sure whether one can trust oneself to play according to one's own self-interest. There is a commitment problem inherent in the majority-rule bargaining game. The proposer who forms a majority must create a proposal which maximizes his own utility and which will give coalition members sufficient incentive to vote yes. Members who are excluded from the majority coalition will do very poorly since there is no incentive for the proposer to distribute any of the cash to their districts, nor is there incentive to accommodate their preferences in the public good choice. Consider a slightly different game in which the proposer posts his proposal followed by a comment period before putting it to a vote. During the comment period, most excluded members will be able to say, “Dear Proposer, please consider a slightly altered proposal in which I am included. We would both be better off if you excluded a different member and gave me a small amount of the cash and moved the public choice just a little in my direction.”. However, the proposer knows that even though such statements from excluded legislators are not lies, he does know that the very people suggesting these altered proposals would vote no on them. This is because an excluded legislator can only recommend changes which make both the excluded member and the proposer better off and the only proposals which accomplish that are those which give the excluded member less than her continuation utility. That is, less than the expected utility she would receive by allowing the current proposer to fail. In other words, excluded players cannot credibly commit to vote YES on a proposal which would make them better off than the current standing proposal but worse off than allowing the current proposal to fail. And in equilibrium, the proposer understands this.

This commitment problem inherent in the majority rule bargaining game allows for the possibility that a voter may be better off being represented by a legislator with different preferences. The question then becomes, should voters try to elect someone more extreme than themselves or more moderate? In general, of course, it depends. First it will depend on the voter’s value of $\alpha$. That is how much weight they place on the public policy outcome of the legislative bargaining session compared to the private wealth distribution question. Preliminary numeric results confirm what is perhaps common intuition with respect to $\alpha$: The more weight the voter places on the public choice...
in her utility function, the more likely, she may vote for a representative whose preferences are even more extreme than her own. However, there is a trade-off here even for the voter who only cares about the public policy choice. Second, it will depend on the composition of the legislature. The voter after all can only elect one member. The ultimate question is, how does her optimal choice of representative ideal point depend on the other $n-1$ members elected from other districts? This is complicated. I present numeric results from 3 different types of 5-player legislatures, where the value of the 4 members not from the voter’s district are known.

The bargaining game and its equilibrium characteristics are described in detail in my paper “Polarized Coalitions in a Legislative Bargaining Game”. In that paper, I described two versions of a legislative bargaining game, a finite horizon and an infinite horizon version. Installing either one of these as a subgame in the broader election model described here presents problems. In the 2-period game there is a unique pure-strategy subgame perfect equilibrium. This fact makes the forward-looking computation for voters in an election simulation relatively easy to make. However, the two-period equilibrium has a serious flaw. For most games (configurations of the legislators’ ideal points) the player with the median ideal point is included in all the coalitions formed by every proposer. This results in a huge bonus for the median player. Voters electing representatives to such a bargaining session would almost always want their representative to be the median player. The pull toward the median in that game is so strong that even voters with extreme preferences for the public good tend to want to elect a representative with median preferences since they are more easily able to influence the final policy choice by virtue of being included in the winning coalitions approximately twice as often as non-median representatives. This in itself would not have been a serious flaw if there was good reason to believe in the precise form of the 2-period equilibrium. However, in Guse (2005) the two-period game was solved not because one should be interested in that game per se, but because it illustrated clearly the incentives legislators have to form their coalitions starting from one extreme (polar) end of the ideological spectrum or the other. The very fact that the median player is included in every coalition in that game makes that precise form of equilibrium unsustainable for any horizon length longer than two periods. Therefore in this paper, I focus on stationary equilibria in the infinite horizon game. The drawback here is that, unlike the 2-period game, there is no known analytic formula to describe any equilibrium. Instead we are left with numeric solutions and then only for 5-player games. With the current algorithm I use a 5-player infinite bargaining equilibrium takes several seconds to solve. The solutions are in the form of distributions over outcomes. This level of computational intensity in the subgame is multiplied when one considers voters in the preceding election stage. For example, just to calculate the optimal choice of representative ideal point in a particular legislature whose other members’ ideal points are already known may take several minutes of computational time depending on the
desired precision. This is because for each candidate representative under consideration, a new legislative subgame equilibrium must be found.

4. Optimal Choice of Agent Ideal Point

I show the results from three sample legislative environments. In each case, the basic question is as follows. What are voter preferences for the ideal point of a legislative representative, given the ideal points of the other 4 legislators with whom the representative will have to bargain and given the ideal point of the voter herself as well as the weight the voter puts on the public policy outcome versus the private redistribution outcome. In other words we want to characterize 

\[ x^*_r(i)(x_v(i), \alpha, \{ x_r(j) \}_{j \neq i}) \]

defined as follows.

\[
x^*_r(i)(x_v(i), \alpha, \{ x_r(j) \}_{j \neq i}) = \underset{x_r(i)}{\text{argmax}} \quad E \left[ (1 - \alpha) y_i - \alpha (x_v(i) - x)^2 \right]
\]

where \( y_i \) is the share of the available surplus allocated to district \( i \) in the final legislative outcome, \( x \) is the value of the public policy in the final legislative outcome, \( x_v(i) \) is the \( i \)th district’s voter’s ideal point and \( x_r(i) \) is the \( i \)th district’s representative’s ideal point. \( y_i \) and \( x \) are random variables whose distributions depend on \( \{ x_r(j) \}_{j \in N} \). That is, they depend on the voters own representative’s ideal point, \( x_r(i) \), as well as the ideal points of the other representatives which the voter cannot choose. As a practical matter in the following examples, the manner in which the set of representative ideal points, \( \{ x_r(j) \}_{j \in N} \), affects the distribution of \( y \) and \( x \) is specified by the equilibrium in the legislative bargaining game which, again, is found numerically. A detailed description of how the legislative equilibria are calculated can be found in Guse (2005), particularly, section 4, “Infinite Horizon Legislative Bargaining”.

The three examples below are designed to show a few defining types of legislatures. In each example the value of the ideal points of four legislators is held fixed and we consider the consequences to a voter from a fifth, “home district” of electing a range of candidates. In the first example, I look at \( \{0.0, 0.33, 0.66, 1.0\} \) so that the other four legislator’s ideal points are spaced evenly across the policy spectrum and reach to both extremes. In the second example, I look at \( \{0.35, 0.45, 0.55, 0.65\} \) so that the other four legislator’s ideal points are evenly spaced with respect to each other but there is room for the fifth, home district, candidate to be more extreme than any of them. In the third example, I have set the other 4 four legislative representative ideal points in a skewed arrangement to \( \{0.0, 0.1, 0.2, 1.0\} \). The interested reader can generate the simulated data series for any configuration of other 4 ideal point by running the program available for download from my website.\(^7\)

\(^7\) Just go to http://home.wlu.edu/~gusej/research/workingPapers.html, click on “ELECTING IDEOLOGICAL REPRESENTATIVES” and look for the infinite horizon version of the “Voter Best Response / Voter Utility”. (The finite horizon
For each example, I show three separate diagrams. In each case the first diagram shows the \textit{expected cash share} for each of 101 different home district representative ideal points, 0.0 through 1.0 in increments of .01. More precisely a point on the graph in these diagrams is a pair \((x_{r(i)}, E y_i)\) showing how much cash on average would be returned to district \(i\) if a representative with ideal point \(x_{r(i)}\) were elected. The expectation is taken over the distribution of legislative subgame equilibrium outcomes found by the numerical algorithm. If one considers voters whose value of \(\alpha\) is zero, one may interpret these graphs as depicting their expected utility as a function of elected representative ideal point.

The second diagram shows, for each configuration of other 4 legislative ideal points, the expected value of the other component of voter utility: \textit{ideological loss}. Each of these diagrams shows the expected loss for the same 101 candidate values of home district representative ideal points. Unlike expected cash share, expected ideological loss depends on the ideal point of the voter in the home district. Hence each of these diagrams shows graphs for 6 different values of voter ideal point. Whereas expected cash could be interpreted as expected utility for voters whose \(\alpha\) is zero, these graphs may be thought of as the expected utilities for voters whose value of \(\alpha\) is one. To get a feel for the expected utility of a voter with an intermediate value of \(\alpha\), one must imagine adding these graphs to the graph for expected utility linearly weighted by \(\alpha\) and \((1 - \alpha)\).

The third diagram for each example is the \textit{best response function} itself - the \(x_{r(i)}^*(x_{v(i)}, \alpha, \{x_{r(j)}\}_{j \neq i})\) defined above. These graphs plot, for each home district voter\(^8\), the optimal choice of representative ideal point. Since this determination also depends on \(\alpha\), multiple graphs are presented in these diagrams - one for each of several values of \(\alpha\).

4.1. \textbf{Example 1. Other ideal points } \{0.0, 0.33, 0.66, 1.0\}. In this example, I consider 101 possible legislative subgame equilibria based on legislatures with an ideal point composition of \{0.0, 0.33, 0.66, 1.0, \(x_{r(i)}\)\}. Values of the home district representative ideal point \(x_{r(i)}\) range from 0.0 to 1.0 in increments of 0.01.

Figure 4 shows the expected cash share going back to the home district. We see that a representative with an ideal point of 0.53 would maximize the home district’s share of the cash to be distributed in the legislative bargaining game. Note that expected cash decreases in both directions from that maximum points. Consistent with the finding in Guse (2005), players who take moderate position in the legislative bargaining game have higher continuation utilities and higher probability of being included in the majority. Hence it is not surprising that the simulations found that a value near the middle in the cash-loving voter’s best choice, since they will have the best

\(^8\)101 values of voter ideal points from 0.0 to 1.0 in increments of .01

version assumes a two-period legislative bargaining subgame, but for reasons stated above, I have decided that simulations run under that assumption are less interesting. Though the finite horizon version has the advantage that the equilibria in the legislative subgame can be calculated in less than a millisecond instead of several seconds.)
bargaining power in that sense. It is remarkable - surprising to me - that the graph appears to be more or less continuous around the values of 0.33 and 0.66. The home district’s representative is qualitatively changing roles in the legislative game, being included in different coalitions as his ideal point crossed these thresholds. A formal proof that the expected cash share is continuous at these points would no doubt be enlightening, but as stated earlier I have not written down an analytic description of equilibrium in the infinite horizon bargaining game. Another thing to note about this graph is that it appears to be not quite symmetric, even though the distribution of other four ideal points is. Indeed, there is no reason to believe that the graph should not be symmetric. The extent to which it deviates from perfect symmetry is explained by precision parameters in the equilibrium search algorithm.\(^9\) Note, below the horizontal axis in Figure 4 is an indication of which “type” of equilibrium prevailed according to the search algorithm for each value of representative ideal point.\(^{10}\)

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\(^9\) For example as the algorithm searches for a stationary equilibrium, it looks to a stopping criterium which is based on the distance a candidate solution to a nonlinear system of equations is from a true solution. The tolerance for allowing the algorithm to stop is set to some strictly number.

\(^{10}\) In Guse (2005), I explain how in the 5-Player infinite horizon bargaining game - the one assumed here, there is a set of four support profiles one of which always forms the basis of any stationary equilibrium. In that paper I labeled those four support profiles according to how many left (L) and right (R) polar coalitions were in the support of legislators mixed strategies. Hence the names 3L1R, 3L2R, 2L3R, and 1L3R.

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Figure 4. Expected Cash Share secured for home district as a function of the home district representative ideal point when the other 4 districts representative ideal points are fixed at \(\{0.0, 0.33, 0.66, 1.0\}\).
Figure 5 shows for various voters the expected (unweighted) ideological loss of choosing representatives with ideal points on the horizontal axis. We see, for example, that voters with ideal points of either 0.8 or 1.0 who care only about the choice of public policy made in the legislative bargaining game would be best off choosing a representative with an ideal point of 1.0. In general these graphs largely, but not uniformly, confirm the intuition that for voters who only care about the public policy outcome in the legislative bargaining game, the best thing is to elect someone more extreme than oneself in order to pull the final policy decision toward one’s own ideal point, a “centrifugal” force on voters’ optimal choice, if you like. Note, however, that relatively moderate policy-centric voter with an ideal point of .4 wants to elect a candidate who is even more moderate with ideal point of .44, suggesting the presence of a “centripetal” force.\footnote{The use of these terms in the context of political theory can be traced to Cox (1990). However, the models analyzed in that paper were purely electoral - no subsequent legislative bargaining game. Hence if the simulated results are correct, this represents a novel source of such forces.}

Figure 6 shows the optimal choice of legislator’s ideal point as a function of the home district voter’s own ideal point for various values of “α” - the relative weight on ideological losses in the voter’s utility function. Note that for sufficiently low values of α, the best response of all voters is to simply choose the representative who can maximize the district’s share of cash. This is the representative with an ideal point of about .54 in this case which is confirmed by figure 4. As α increases there is more elasticity in the best response function. In all cases, the best response
functions are weakly increasing in voter ideal point - the greater the ideal point of the voter, the greater the optimal choice of ideal point for the representative.

It is worth noting that even when \((\alpha = 1)\), the cash maximizing representative is the optimal choice for a significant (but fairly narrow) band of voters. This was somewhat surprising to me, but begins to make sense when one accepts the presence of a centripetal force on voter incentives observed above in Figure 5. The ideological loss component of the voter utility function appears to give moderate voters an incentive (centripetal) to elect representatives are “more moderate” - close to the median value, for example. On the other hand the ideological loss seems to have the opposite effect (centrifugal) on relatively extreme voters giving them incentive to elected even more extreme representatives. I would be hesitant to say with certainty on the precise origin of these effects, but I my speculation would run along these lines. Consider that according to the numirical results discussed in Guse (2005) having the representative ideal point toward the center of the legislative distribution of ideal points does two closely related things. First, it increases the probability that that the representative will be included in the majority coalition. This may partially explain the effect. Just having a moderate included more often in the majority, one might think, should moderate the overall distribution of policy outcomes. But why? I believe this can be explain in the context of a second phenomenon. We know from Guse (2005) that only “polar” or near polar coalitions form in equilibrium. Hence when the middle guy is included in the majority his coalition partners will tend to be all to the right of him or all to the left of him.\(^{12}\) Since the policy choice will always be the mean of the coalition members’ ideal points in any particular legislative outcome, a more moderate middle man will reduce the overall variance of the policy distribution by pulling both the left polar coalitions and the right polar coalitions toward the center. Moreover the increased probability of inclusion for the more moderate middle guy re-enforces this effect. Relatively moderate voters effectively care about keeping the variance of the policy distribution low because of their convex loss functions. While this is true for relatively extreme voters as well, they are more willing to trade off higher variance in exchange for pulling the the expected value of the policy outcome toward their ideal point.

\(^{12}\)In fact, the only coalitions arising in equilibrium which have the median legislator as a member are pure polar coalitions - coalitions that can be construction by starting on one end of the ideological distribution and moving toward the center until a minimum majority of member is included. Again see Guse (2005) for details.
4.2. Example 2. Other 4 Representative Ideals = \{0.35, 0.45, 0.55, 0.65\}. In this example I consider 101 possible legislative subgame equilibria based on legislatures with an ideal point composition of \{0.35, 0.45, 0.55, 0.65, x_r(i)\}. Values of the home district representative ideal point \(x_r(i)\) range from 0.0 to 1.0 in increments of 0.01.

There are many similarities. The expected cash share going back to the home district as a function of representative ideal points shown in Figure 7 tells a familiar story. The best thing for a cash-loving (\(\alpha = 0\)) voter is to elect someone with an ideal point around .5 and the penalty of doing otherwise should be symmetric around that point. Again, the lack of perfect symmetry in Figure 7 belies precision issues in the equilibrium search code, but the general inverted u-shape is quite clear.\(^{13}\) Figure 8 also tells a very similar story as Figure 5 did from Example 1.

\(^{13}\)An interesting note about the apparent discontinuities in Figure 7 is they occur not when the representative ideal point cross over the other four ideal points. Instead two of them occur when the type of equilibrium changes. In Guse (2005), I explain how there are four types of equilibria which one typically expect to arise in the infinite horizon game represented by four “support profiles”, 3L1R, 3L2R, 2L3R, and 1L3R. I have note below 7 which type of equilibria the algorithm settled on for each representative ideal point.
Figure 7. Expected Cash Share as a function of home district representative ideal point when other four districts have representatives with ideal points \{0.35, 0.45, 0.55, 0.65\}.

Figure 8. Expected Ideological Loss component of voter expected utility when other four legislators have ideal points \{0.35, 0.45, 0.55, 0.65\}.
The greatest contrast between voter incentives in Example 1 and this example makes itself apparent in the best response graphs shown in Figure 9. Here we see that voters are quicker to polarize - that is to respond to the centrifugal forces electing a candidate more extreme than oneself. This can be seen in the fact that the range of voter ideal points for which it is optimal to elect the representative whose ideal point is .5 has narrowed for all values of $\alpha$. Again I think that a consideration of the mean variance trade-off in the policy outcome distribution might explain this. Since the distribution of ideal points in the legislature is tighter compared to that in Example 1, variances in the policy outcomes are going to be lower. This might lower the marginal cost in terms of variance of pulling the mean of the policy distribution toward ones own ideal points. Again let me stress that this speculation based on observations of patterns which emerged in the numerical analysis analyzed in Guse (2005).

![Figure 9. Optimal choice of Representative as function of voter ideal point and $\alpha$ when other four representatives in the legislature have ideal points \{0.35, 0.45, 0.55, 0.65\}](image)

### 4.3. Example 3. Other 4 Representative Ideals = \{0.0, 0.1, 0.2, 1.0\}. In this example I consider 101 possible legislative subgame equilibria based on legislatures with an ideal point composition of \{0.0, 0.1, 0.2, 1.0, $x_{r(i)}$\}. Values of the home district representative ideal point $x_{r(i)}$ range from 0.0 to 1.0 in increments of 0.01.
Figure 1. Expected Cash Share as a function of home district representative ideal point when other four districts have representatives with ideal points \( \{0.0, 0.1, 0.2, 1.0\} \).

Figure 2. Expected Ideological Loss component of voter expected utility when other four legislators have ideal points \( \{0.0, 0.1, 0.2, 1.0\} \).
5. Conclusion

In this paper I have considered voter incentives in a multi-district election game where the winners go on to bargain over both a public policy and the distribution of private goods in a majority-rule legislature. For concreteness, we can think of the public policy question as something like the degree of abortion restrictions which has little effect on the distribution of wealth across districts and the private goods decision as something like federal highway funding “brought home” to the local districts. The legislative bargaining model itself is taken from Guse (2005) where I described how to calculate equilibria numerically and discussed emergent pattern across games (configurations of legislator ideal points) in the setting of a 5-member legislature. In this paper, I discuss simulation results from 3 different scenarios in which a voter in one district, the “home district”, determines his optimal choice of representative when he knows the ideal points of the other 4 district representatives in the legislature. I assumed that voters care both about how many federal dollars are returned to their district as well as the final value of the public policy. This paper concerned itself only with the best response function of voters in this broader election game. No attempt was made at to fully analyse an equilibrium. In particular, I made no account for how voters in other districts might change their choice of representative, given the optimal choice of the home district. That said, a few interesting patterns emerged from this simple analysis of incentives. First, we saw in every example that to the extent voters care about the expected cash share (federal
dollars brought home), they have a strong incentive to moderate their vote. Second we saw that the public policy component of voter preferences can actually have one of two opposite effects. It may give incentive for voters to moderate or it may give them incentive to vote for candidates more extreme than themselves. Moreover, the simulated data suggests that incentive to moderate (centripetal force) is felt more strongly by relatively moderate voters while the incentive to vote extreme (centrifugal force) is felt more strongly by relative extreme voters. I speculated on why this may be by appealing to the polarization results from Guse (2005). Less interesting but still comforting, I found that the best response functions are weakly increasing; the higher the value of a voter’s ideal points, the higher is the optimal choice of representative ideal point.

6. References


