Beyond strong sustainability

Dina Franceschi1 and James R. Kahn2

1Department of Economics, Fairfield University, Fairfield, CT
2Director of Environmental Studies and Department of Economics, Washington and Lee University, Lexington, VA

Key words: sustainable development, natural capital, environmental services

SUMMARY

Since the Brundtland Commission’s delineation of the term sustainable development in 1987, virtually every country has incorporated the terms sustainability and sustainable development into their planning vocabulary and criteria for decision-making. However, many issues remain unresolved. Broad and sweeping references to sustainability and sustainable development do not necessarily translate into implementable policies to achieve these goals. In particular, unresolved issues include developing an understanding of how one sector of the economy can contribute to the sustainable development of the economy as a whole and the role of ecological resources in sustainable development. Our paper provides an initial conceptual examination of these questions by folding mining and ecological quality into the sustainability discussion. We use the Brazilian Amazon as an application of our sustainable development model.

INTRODUCTION

The term and concept of sustainable development has received much attention since the late 1980s, when it became the global theme for a long-term perspective on economic growth. The definition of the term has been moulded to fit different situations since then (see Pezzey, 1989 for 35 definitions), but has mostly retained the original thrust behind which sustainable development is driven. Generally, the interpretation relates improving the prospects of the current generation without reducing the prospects of future generations (World Commission on Environment and Development, The Brundtland Report, 1987).

While policymakers attempt to operationalize the concept, the debate over the degree to which sustainability can be implemented continues. Some believe policy should adhere to a strict interpretation, e.g. strong sustainability, while others suggest a more relaxed version, i.e. weak sustainability (Cabeza Gutes, 1996; Hediger, 1999). In particular, Cabeza Gutes (1996) writes on the concept of weak sustainability and its relationship to fundamental growth theory with exhaustible resources. He finds a troubling limitation in the applicability of the concept of weak sustainability that allows substitutability between various types of capital (specifically natural and man-made capital). The potential for sustainability concluded in the literature (Solow, 1974; Stiglitz, 1974; Hartwick, 1977, 1993; Dasgupta and Heal, 1974; Pearce and Atkinson, 1993) depends on the substitutability cornerstone. Later, the literature on strong sustainability (Stern, 1997a; Hediger, 1999) relaxes this restrictive assumption, attempting to reconcile

Correspondence: Dina Franceschi, Department of Economics, Fairfield University, Fairfield, CT 06824, USA. e-mail: dfranceschi@ mail.fairfield.edu

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
the inconsistencies with the notion of weak sustainability.

Cabez de Gutes (1996) also points out that the aggregation of natural capital as an input type also constrains the usefulness of the weak sustainability concept (as seen in treatments such as van Geldrop and Withagen 2000). This limitation arises from the fact that other important links between the environment and the economy, like the need for natural capital to produce manmade capital, or even more crucially, the role of the former as life-support services, are excluded from the concept. One subset of the broader aggregation problem is that weak sustainability treats natural capital as a homogenous category of capital. He later refers to the distinction as critical and non-critical natural capital (i.e. the absorptive capacity of a wetland vs. an iron ore deposit). Collados and Duarte (1999) make a similar argument.

Of secondary, but equally important motivation our study of and interest in Amazonia. Since the 1992 Earth Summit in Rio de Janeiro, the notion of sustainable development has been put on the political map and in every planning document on the planet. As the global community tries to figure out how to most effectively use the Earth’s environmental resources, we must come up with development strategies that not only account for the traditional value of natural resources used in economic production but also those that indirectly contribute to economic production, namely environmental services. Amazonia is teeming with such services which give it value beyond traditional economic measures. Further, Brazil, as well as all of the Amazonian countries, are faced with tough development choices. High rates of unemployment and poverty put increasing pressure on the government and private sectors to find work and economic growth for the people of the region. Because Amazonia accounts for a large portion of the total land area in countries such as Brazil (about 50%), future economic development depends on the efficient use of its natural resource base (e.g., mining). It is important for domestic growth and global concern to find policy that allows development but maintains environmental preservation. For this reason, we advocate scrutinizing the notions of sustainability and their application to unique environmental areas such as Amazonia.

TRADITIONAL APPROACHES

Neoclassical growth and development models

Neoclassical growth and development models tend to focus solely on problems of artificial (human-made) capital accumulation (Stern, 1997b). These models developed rules for allocating current output between consumption and investment in artificial capital. For example, the classic Harrod–Domar model of development sets up a system of economic growth equations based on the full employment of capital and labour, where the steady-state rates of growth of capital, labour and output are equal. The conclusions of the model, given exogenous factors, are that the warranted rate of growth is equal to the ratio of the propensity to save to the capital-output ratio. Variations on the paradigm include, Solow’s factor substitution model which resolves the same question but with added flexibility in inputs (Kaul I.), and Kaldor’s saving rate adjustment model which allows flexibility in the saving mechanism to compensate for disproportional labour force growth. Recall, however, that the majority of the classic growth models examined savings and investment from purely the capital-profit or labour-wage standpoint.

The policy implications of these models are that development can be viewed as a process of artificial capital accumulation and of removal of obstacles to the artificial capital accumulation process. Poverty of nations was viewed as a self-perpetuating process, a vicious cycle of poverty which is depicted in Figure 1. In this view of economic development processes, low income and high current consumption needs lead to low savings, low investment in artificial capital and a lack of artificial capital formation. The low stock of artificial capital implies a continually low marginal product of labour, with a correspondingly low level of income. In fact, since human-made capital depreciates, capital stocks may fall over time due to insufficient investment to counter depreciation, and as a result, income may be falling over time. In this perspective on development, the solution to the problem is simple: inject capital and break the cycle of poverty. This was essentially the basis of the international development programmes of the 1960s and 1970s. Unfortunately, in many places
this development approach was largely unsuccessful. While rapid population growth, civil war, AIDS and government corruption may have been important factors in the lack of economic development in some countries, another important factor is that development policy may have focused too much on the accumulation of human-made capital, to the exclusion of consideration of other types of capital, particularly natural capital.

Figure 2 illustrates the shortcomings of these traditional models by showing how environmental degradation can effect the cycle of poverty. Low income and high current consumption needs lead to environmental degradation from over exploitation of renewable resource systems, causing their (often irreversible) collapse. Also, inefficient production technologies lead to high emissions of waste products, causing urban environmental problems and contamination of both surface waters and aquifers. This environmental degradation also lowers the marginal product of labour. A decrease in the marginal product of labour is particularly important in the rural areas of developing countries, where desertification, deforestation, fishery collapses and soil erosion/fertility loss lead to losses in agricultural productivity. In addition, the human health consequences of air pollution and drinking water contamination makes labour less productive and generate costs such as those associated with the treatment of health problems. Finally, environmental resources provide ecological services such as maintenance of climate, biodiversity, nutrient cycling, waste assimilation, primary productivity, maintenance of atmospheric chemistry, soil formation, protection of water resources and flood protection, all of which are important to basic life processes, quality of life and economic productivity.

Sustainability and exhaustible resources

The conventional treatment of exhaustible resources in terms of sustainability dates back to the early works of Malthus and Ricardo, where resource scarcity and diminishing returns domi-
nated the analysis. These models focused on finite resource supply and whether continued economic growth was possible as resources became more scarce. Work by Barnett and Morse (1963) built on these ideas by emphasizing the inevitable development of good substitutes for the resources that become characterized by increasing scarcity. Utilizing a model based on the free substitution of inputs (labour, artificial capital and exhaustible resources), Barnett and Morse found that technological innovation in discovery, extraction and production techniques lowered the costs of extractable resources, greatly outweighing the cost-increasing effects of depletion. These influential findings largely refuted the previously held notion that Malthusian limits to growth existed and would constrain or collapse current consumption and development rates. Barnett and Morse's conclusion has served as the point of embarkation for further exploration of the relationship between sustainability and exhaustible resource use, including works by Hartwick (1977 and 1993), Mikesell (1997), Pezzey (1989), Pezzey and Withagen (1998), Tilton (1996) and Vincent et al. (1997).

The assumption of substitutability between inputs is not an inconsequential one. The fundamental premise underlying the work of Barnett and Morse is the interchangeability or perfect substitutability between labour (human capital), man-made capital, and extractable natural resources as inputs to the production process. Two issues regarding substitution to achieve sustainability in these models are of concern.

The most important issue is that the models do not incorporate the environmental resources as inputs to the production process. As discussed earlier, environmental resources provide an array of important services to a variety of different production processes, day-to-day living and ecosystems (Collados and Duane, 1999). However, the inclusion of environmental resources into the production function is not in itself sufficient to rectify the shortcomings of these economic models of sustainability. In addition, one must consider the notion of complete substitutability among the different types of capital (see Pearce and Warford, 1993). Kahu and O’Neill (1999) argue that at the large scales defined by the current level of economic activity, artificial capital cannot provide an adequate substitute for environmental resources, as it is completely infeasible for human activity to provide the ecological services at the level of natural activity. For example, although artificial capital can be used to process point source wastes in a sewage treatment plant, it would be completely infeasible for all the non-point sources of nutrients (both natural and anthropogenic) to be processed by treatment plants. The importance and economic value of wetlands has been realized for this very reason. It should be noted that scale is not the only source of concern. Ecological complexity argues against complete substitutability between environmental and other types of capital (Kahu and O’Neill, 1999). The Amazon forest for example, one of the last largely intact tropical rain forests, provides the second most efficient carbon sink for greenhouse gases in the world (second to oceanic surfaces). Its ability to sequester carbon is attributable to both its size and its rich ecological complexity.

Further, the traditional neoclassical approach (which suggests the mitigation of scarcity associated with exhaustible resources) is based on the premise that the market can allocate supply and demand and create appropriate incentives through the pricing system. In the classic literature of resource scarcity, including the work by Barnett and Morse (1963), as an exhaustible resource or mineral resource base becomes depleted, prices rise. The rise in price is primarily the response of increased costs of extraction of the relatively less abundant source (relative in the sense of the deposit being less rich than deposits of the past). As price of one product rises, either firms find better extraction technology to control costs or buyers switch to another product that can satisfy their needs at a lower price. In the case of most environmental resources, no comparable market mechanism exists to send appropriate signals or create incentives to mitigate scarcity. Most often ecological systems do not have prices associated with their services, because they are external to markets. The public goods (and often open-access) nature of ecosystems often precludes sustainable long-term usage. Again, the fact that these systems are exceptions to the classic market mechanisms heightens the need for their explicit inclusion in the formula for sustainability.
A REVISED TREATMENT OF SUSTAINABILITY

Solow (1974), Hartwick (1973), Stiglitz (1974), Pezzey (1989, 1998), and others have contributed to the literature dedicated to finding the optimal rate of exhaustible resource extraction for intergenerational sustainability. Hartwick, in particular, has derived a saving and investment rule for economics utilizing exhaustible resources. The Hartwick model and its results focus on the generation of sustainability through a replacement mechanism. The rule defines the level of investment and savings which must be generated by the current generation to replace the natural capital (exhaustible resource stock) they have consumed, in order to maintain a constant level of consumption for future generations. (Hartwick, 1977 and 1994)

Using conventional production and consumption functions, Harwick defines society’s reinvestment rule as a level of investment set equal to a chosen proportion (\( \alpha \)) of the rents associated with the production of exhaustible resources. With extraction costs assumed to be zero, rents would equal the product of resource extraction (\( R \)) and the marginal product of resource flows (\( F_R \)). The reinvestment rule would then be given by equation (1).

\[ \dot{K} = \alpha (RF_R) \]  

The model then employs Hotelling’s efficiency rule for exhaustible resources to find (see Appendix):

\[ \dot{C} = (1 - \alpha) F_R \dot{R} \]  

if \( \alpha = 1 \), then \( \dot{C} = 0 \).

Hartwick concludes that, as long as the current generation is reinvesting exactly the rents derived from the extraction of the exhaustible resource, consumption can be maintained at a constant level over the time horizon chosen. However, this equation merits further discussion. Since \( F_R \), the marginal product of resource flows, is positive and \( 0 \leq \alpha \leq 1 \) the sign of \( \dot{C} \) will depend on the sign of \( \dot{R} \). If the stock of resources is finite (and technology is assumed constant), then \( \dot{R} \) must eventually become negative. Therefore, the greatest possible value of \( \dot{C} \) is zero, which will occur when \( \alpha = 1 \) (when all the rents are reinvested in artificial capital).

In Hartwick’s model, \( \dot{C} \) cannot be increasing. However, if one allowed the stock of resources to increase (through technological innovation), then \( \dot{R} \) could be increasing and the reinvestment rule would change, as \( \alpha = 0 \) would maximize \( \dot{C} \).

Of course, if technological innovation required investment in artificial capital, then the optimal proportion of reinvestment would be somewhere between zero and one, exclusive of the extreme values.

In recent years, the Hartwick approach has been scrutinized as well as extended. This paper does not refute the limitations or contributions of the Hartwick rule. Instead, we bring attention to omissions in the original model that are important for application of the model. Obviously, the model does not incorporate environmental resources which, as discussed earlier, have important implications for sustainability. Environmental resources can be added to the Hartwickian approach which will add considerable insight to the understanding of the necessary and sufficient conditions for sustainability in regions with rich ecological complexity such as Amazonia.

Environmental resources can be incorporated by taking the same initial construct with the addition of a term that relates the extraction of the exhaustible resource to the depletion of ecological services. We start by assuming that the environmental resource stock is proportional to the exhaustible resource stock. This assumption in no way compromises the results of the model and is applicable in the analysis of sustainable development policy in regions with similar characteristics to our area of interest, Amazonia. As the mineral resource stock (S) is extracted, it gets smaller, more land is used and greater degradation occurs, thus causing the environmental resource stock (E) to get smaller as well. Also, ecological services are some function of the environmental resource stock. If ecological services are modelled as enhancing utility directly, consumption can now be defined as production plus ecological services minus the change in capital.

\[ E = BS \]  

\[ C = Q + pE - \dot{K} \]  

Similar mathematical analysis, using the reinvestment rule (1) and Hotelling’s efficiency rule.
results in a potentially non-sustained rate of consumption over time.

\[
\dot{C} = (1 - \alpha) F_K \dot{R} + \rho \dot{E}
\]  

(5)

If \(0 \leq \alpha < 1\) and both the time derivatives of \(R\) and \(E\) are negative, then the time derivative of consumption must be negative. Following the Hartwick rule of total reinvestment of resource rents, where \(\alpha = 1\), the change in consumption moves with a change in the environmental stock.

- if \(\dot{E} > 0\), then \(\dot{C} > 0\)
- if \(\dot{E} < 0\), then \(\dot{C} < 0\)

Consumption can actually be enhanced by a growth in ecological services (e.g., allowing cleared forest land to revert back to original forest, increasing global carbon sequestration capacity) or conversely, diminished by a negligence in environmental protection (e.g., continued promotion of harvesting and clearing of forest). The results become a bit ambiguous when the reinvestment of rents rule is relaxed. Depending on the relative magnitudes of the two terms, the change in consumption could be negative or positive.

The interpretation of equation (5) places the sustainability debate in context. At one extreme would be the ‘technological optimists’ who would argue that \(\dot{R}\) is positive, that both \(\dot{R}\) and \(F_K\) are large, and that this would compensate for a negative \(\dot{E}\). At the other extreme would be the ‘environmental pessimists’ who would argue that \(\dot{E}\) is negative, that \(\dot{R}\) cannot be positive due to the finiteness of crustal mineral deposits, and that \(\dot{C}\) must be negative.

A less extreme approach would recognize the sign of \(\dot{C}\) as a policy choice. We could invest in research and development and artificial capital to correct for the problems of production (for instance, costs associated with the treatment of declining health of workers). Alternatively, as the notion of sustainable development suggests, we could protect the environmental resources that provide ecological services, such as maintenance of climate, biodiversity, nutrient cycling, waste assimilation, primary productivity, and maintenance of human capital, as a preventative measure. However, if the sign of \(\dot{C}\) is a policy choice and sustainability is the goal, then sustainability must be pursued with an active policy agenda

and not simply left to market forces, as the market forces which alleviate scarcity of mineral resources are not applicable to environmental resources.

A similar result with parallel implications occurs when the representation of environmental resources enters the model through the production function, as opposed to entering into consumption directly. Production in this case is a function of the capital flow, exhaustible resource flow and environmental resource stock.

\[
C = F(K(t), R(t), E(t))
\]  

(6)

\[
\dot{C} = (1 - \alpha) F_K \dot{R} + F_E \dot{E}
\]  

(7)

The possibility of declining consumption again occurs, determined by the magnitudes of the terms. In any case, the change in consumption over time is either improved or diminished by the marginal contribution to production of the change in ecological services over time. Assuming that the marginal product of the environmental asset is positive, when \(\alpha = 1\), the change in consumption again moves with the change in the environmental stock.

- if \(\dot{E} > 0\), then \(\dot{C} > 0\)
- if \(\dot{E} < 0\), then \(\dot{C} < 0\)

One can gather considerable insights by looking at Equations (5) and (7). Policy must strive to make the sum of both terms of these equations positive. This may not seem possible, given the initial assumptions of the model, but the assumptions can be relaxed to allow for technological change. First, technological innovation can change the sign of the time derivative of \(R\). Although the crustal abundance of mineral resources cannot be increased, technological innovation can increase the availability of the resource stock \(S\) over time, implying that \(R\) need not be declining over time. \(R\) can also be increasing over time due to the judicious use of renewable resources, including the sun’s energy. Second, the time derivative of the environmental resource is actually equal to \(-\beta R\). The exhaustible resource stock and flow relationship, \(S(t) = S(t) R(t)\) combined with equation 4 leads to \(E(t) = -\beta R(t)\) where \(\beta\) measures how much extractive resource usage depletes the stock of environmental resources. The more that technological...
Innovation can reduce the environmental impact of extractive resource use \( (\beta) \), the smaller the second term of equations (6) and (8) and the less it would offset a potentially positive first term. It is important not to lose sight of the substitutability arguments that were made above. If artificial capital and extractive resources are poor substitutes for ecological services, then as environmental resources decline, \( E \) will become very large, and consumption will inevitably have to decline.

The role of the magnitude of \( E \) is critical in determining the sign of \( \hat{C} \). Figure 3 shows the total product function for environmental services, where the slope of the function is equal to \( E \). As with other inputs, diminishing marginal productivity is expected, and is reflected in the slope of the function in Figure 3. Note that if ecological services are a monotonic function of environmental resources, \( E \) could represent either ecological services or environmental capital in this graph.

Two aspects of this total product function are of interest. First, at high levels of ecological services, the losses associated with a small diminution of ecological services are also likely to be small. In fact, if increases in ecological services become redundant at high levels of ecological services, the slope of the total product function may equal zero. Second, there may be a threshold below which production falls dramatically with further reduction in ecological services. This threshold is likely to exist because of the complexity and nonlinearity of ecological behavioral functions (Kahn and O’Neill, 1999).

Both characteristics have important implications for equations (5) and (7). If \( \hat{E} \) is negative, the product of \( F_1 \) and \( \hat{E} \) will be small in the vicinity of \( E_2 \) and large in the vicinity of \( E_1 \). This illustrates the underlying cause of the assertion that a non-negative \( \hat{C} \) is more likely when the level of ecological services is high than when the level of ecological services is low.

**IMPLICATIONS FOR THE SUSTAINABLE DEVELOPMENT OF AMAZONIA**

The theory in the preceding sections can be used as a guide to develop sustainable development policies. Even without an empirical analysis, it is possible to construct guidelines for development based on an assessment of capital stocks in comparison to capital needs. We choose the Amazon region of Brazil as our example, due to the threats to environmental resources associated with the development process and continued exploration for mineral resources in the area. In this region, there is a strong need for a sustainable development process to protect the global environmental resources associated with the rain forest. Further, the tension between the domestic benefit and the global costs of development highlights the importance of sustainable development analysis.

Given the central role of all types of capital in the process of sustainable development, the first step in analyzing this question is to assess capital stocks in the region. Although the Amazon region is large and heterogeneous, characteristics of the capital stocks are shared throughout the region. In general, the region is characterized by low levels of artificial capital, low levels of human capital, abundant extractive resources and abundant (but threatened) environmental capital. In the following examples, we will show how capital augmentation plans could be developed to promote the sustainable development agenda, and how they may need to differ throughout the region.

The first question is how to create the necessary surplus to increase artificial capital and human capital. Since this paper has focused on extractive resources, these will be discussed first. Currently, mineral production in the Brazilian Amazon is a significant source of regional GDP. The world's largest tin mine is in Pitinga, Amazonas, and the world's largest iron mine is in Carajas, Pará. Mining currently accounts for 2% of GDP, and

**International Journal of Sustainable Development and World Ecology**

Reproduced with permission of the copyright owner. Further reproduction prohibited without permission.
there is significant potential to increase the amount of production. In light of the previous discussion of the Hartwick rule and subsequent modifications, an expansion of mining can lead to sustainable development if two conditions are fulfilled. First, the rents from the mining must be reinvested in capital, both man-made and natural. Second, the mining activity must not result in significant damage to the rain forest or river systems. In other words, the expansion of mining could result in a movement towards sustainable development as long as these two conditions of reinvestment and environmental protection are met.

For other types of extractive activities it is more difficult to obtain a surplus, and environmental impacts can be even more devastating. For example, the clearing of rain forest for agricultural field crops leads to permanent loss of rain forest, and is also unsustainable in another dimension. Poor soils, and the breakage of the nutrient cycle associated with removing the forest cover implies unproductive agriculture, with crop production often limited to 2–4 years. Under these conditions, it is not possible for large-scale agriculture to generate a surplus with which to generate funds for investment, and in fact, leads to much more loss of forest per dollar of current GDP or per job generated in comparison to mining.

Of course, sustainable development does not require that the surplus used for capital investment be generated from within the region. These funds for investment can come from other parts of Brazil or from abroad. In fact, the policies which have been developed to create and sustain the Manaus Free Trade Zone represent a transfer from southern Brazil to the Amazon region that results in an increase in the stock of artificial capital in Manaus. Although environmental protection was not the original motivation for the policies, they have resulted in a slowing of deforestation in the area. The state of Amazonas remains at 97% of its original forest cover while other Amazonian states have lost over 20% of their original forest cover.

Our discussion of sustainable development in the Amazon region would not be complete without discussion of subsistence level activities. For example, small farmers in Rondonia are primarily responsible for deforestation in that state (Caviglia, 1999). These farmers are migrants from southern Brazil and utilize traditional field crop methods instead of the sustainable agroforestry techniques employed by Indians and Caboclos. Caboclos are people of mixed Indian and Portuguese descent who have been rain forest inhabitants for many generations, and whose families have been in the rain forest for as long as three or four hundred years. Caviglia shows that the income that can be generated from sustainable agroforestry exceeds that from traditional field crop methods, but these migrant farmers continue to use the unsustainable techniques. Here, an investment in human capital would contribute to sustainable development through the transmission of knowledge of the Indian/Caboclo sustainable techniques to the migrant farmers who only know the unsustainable temperate agricultural techniques of southern Brazil.

CONCLUSIONS

Although the evolution of sustainable development policies must consider a spectrum of microeconomic, macroeconomic, ecological, cultural, institutional and social issues, our paper suggests that a central focus of sustainable development should be the accumulation of human capital, artificial capital and natural capital, while protecting the stock of environmental capital. Environmental capital is critical to future economic growth and both the economic and general health of every nation. As we have shown, past models do not accurately describe how crucial environmental capital is to achieving development that can be maintained over time. In fact, our results show that the inclusion of even the recently delineated strong sustainability criteria falls short in the sustainable development planning and policy process. Simple reinvestment of rents from productive activities into artificial capital is not enough in most cases to support development for the long term. Reinvestment or preservation in the environmental capital stock can be as important for productivity as human or artificial capital.

Empirical testing, comparing the generation of reinvestable rents and environmental protection in several different productive activities, is
desirable to generate implementable sustainable development policy. For instance, in the case of Amazonia, productivity in sectors such as agriculture, cattle ranching, mining, timber harvest and others should be compared. We leave this to future work.

ACKNOWLEDGEMENTS

The authors would like to thank Steve Stewart, Bruce Wilcox and several participants of the ISEE conference in Canberra, Australia for very helpful comments.

REFERENCES


APPENDIX

I

The replication of Hartwick’s results begins with his definitions of consumption and saving.

\[ C = Q - \dot{K} \]

where \( Q = F(K(t), R(t)) \) and \( \dot{K} \) denotes the change in capital over time or investment. The change in consumption over time then, can be defined as,

\[ \dot{C} = \dot{Q} - \dot{K} \quad (i) \]

This is subject to an equation of motion of exhaustible resource flow, defined as,

\[ -\dot{S}(t) = R(t) \quad (ii) \]

and the same capital reinvestment rule as equation (1) in the text,

\[ \dot{K} = \alpha RF_r \quad (iii) \]

Substituting into equation (i), we have

\[ \dot{C} = F_k \dot{K} + F_r \dot{R} - \alpha RF_r - \alpha \dot{R}F_r \]

Using Hotelling’s rule for efficiency in the extraction of exhaustible resources,

\[ \frac{\dot{F}_r}{F_r} = F_k \quad (iv) \]

and simplifying Hartwick finds (The converse of the model has also been proven and can be seen in Withagen and Asheim, 1998),

\[ \dot{C} = (1 - \alpha) F_r \dot{R} \]

Where \( \alpha = 1, \dot{C} = 0 \)

II

Now define the ecological stock to be proportional to the exhaustible resource stock, as mineral resources are extracted from the ground, forest is cut down.

\[ E = \beta S \quad \text{also,} \quad \dot{E} = \beta \dot{S} \]

In this case, the ecological resource stock impacts consumption directly. The consumption equation, a revision of (i), will be,

\[ C = F(K, R) + \beta S - \alpha RF_r \]

(The representation of time is suppressed for simplicity.) Taking the time derivative gives,

\[ \dot{C} = F_k \dot{K} + F_R \dot{R} - \beta \dot{S} - \alpha RF_r - \alpha \dot{R}F_r \]

After substitution of the reinvestment rule (ii), our equation of motion (iii), Hotelling’s efficiency rule (iv), and simplification, the resulting change in consumption equation is equation (5) in the text,

\[ \dot{C} = (1 - \alpha) \dot{R}F_r + \dot{E} \]

This can be interpreted as the change in consumption over time being supported or reduced by the rate of enhancement or depletion of the ecological stock.

III

The third case incorporates the condition of the ecological stock being proportional to the exhaustible stock into the production function. Where production is now defined as,

\[ Q = F(K(t), R(t), S(t)) \]

Using (i) and (iii) we have,

\[ C = F(K, R, S) - \alpha RF_r \]

Taking the time derivative gives,

\[ \dot{C} = F_k \dot{K} + F_r \dot{R} + F_s \dot{S} - \alpha RF_r - \alpha \dot{R}F_r \]

Substituting (iii) and (iv) and simplifying gives a somewhat similar equation for the change in consumption over time as the second case.

\[ \dot{C} = (1 - \alpha) F_r \dot{R} + F_s \dot{S} \]

Or, by substituting (ii) we have the text equation (7),

\[ \dot{C} = (1 - \alpha) F_r \dot{R} + F_s \dot{E} \]

This essentially means that the change in consumption over time moves with the change in the flow of the marginal productivity of ecological services over time. Again, depending on relative magnitudes, the change in consumption over time could potentially be negative, indicating a decrease in utility for society or an unsustainable path.