Ocular Dominance in the Visual Cortex

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Outline

• Biological background
• First layer: Harris-Ermentrout-Small model
• Second layer
• Feedback
• Future directions
Introduction: visual system

- Photoreceptors convert light falling on retina to electrical signal, which is then relayed to the LGN.
- Left half of visual field maps to right half of cortex, and vice versa, preserving some topography.
Ocular Dominance

- The LGN is segregated into left-eye and right-eye layers
- In mature cortex, inputs to primary visual cortex are segregated
- Ocular dominance: significant influence of, say, left-eye over right-eye activation determining a neuron’s response properties
Early and Late Wiring

Early in Development

Mature State
Ocular Dominance Map

Normal macaque V1 (Hubel/Wiesel 1977)
Ocular Dominance Map

Macaque V1 where one eye is deprived through the critical period

- Monocular deprivation experiments suggest activity-dependent competition between left and right eye afferents.
Layers of the Visual Cortex

Axons and dendrites from various layers

To other parts of the brain

Pyramidal cell

Descending nerve fibres

White matter
Model for OD Development

V1 layer 4

LGN left eye

LGN right eye

\[ w_L(\xi) \]

\[ R(\xi) \]

\[ I_L \]

\[ I_R \]
Hebb’s Rule

Neurons that fire together wire together
Hebb’s Rule

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Neurons that fire together wire together

Needs a normalization constraint of some sort (e.g. competitive mechanism)
Synaptic Strength

\[
(1 - w_R) \quad \xrightarrow{K^+(n_R, V, I_R)} \quad w_R \quad \xleftarrow{K^-(V)}
\]

\[
K^+(n_R, V, I_R) = n_R \cdot V \cdot I_R \quad \text{Hebb's rule}
\]

\[
K^-(V) = \beta_1 \cdot V
\]

\[
\frac{dw_R}{dt} = n_R \cdot V \cdot I_R \cdot (1 - w_R) - \beta_1 \cdot V \cdot w_R
\]

V = postsynaptic activity

\(n_R\) = amount of trophic factor
Lateral Interaction

\[ V(\xi) = \int G_1(\xi - \xi')(I_R(\xi')w_R(\xi') + I_L(\xi')w_L(\xi')) \, d\xi' \]
Averaging Input Correlation

Hebbian term for $w_R(\xi)$ is $V(\xi)I_R(\xi)$

$$V(\xi) = \int G_1(\xi - \xi')(I_R(\xi')w_R(\xi') + I_L(\xi')w_L(\xi'))\, d\xi'$$

$$\langle I_R(\xi)I_R(\xi') \rangle = \sum_{k \in \mathcal{K}} p(k)I^k_R(\xi)I^k_R(\xi') = C_{RR}Q(\xi - \xi')$$

$$V(\xi)I_R(\xi) = \int G_1(\xi - \xi') (C_{RR} w_R(\xi') + C_{RL} w_L(\xi'))\, d\xi'$$
Equation for Weights

\[
\frac{dw_R}{dt} = n_R \cdot V \cdot I_R \cdot (1 - w_R) - \beta_1 \cdot V \cdot w_R
\]

\[
\frac{dw_R}{dt}(\xi) = n_R(\xi) \left( \int G_1(\xi - \xi')(C_{RR}w_R(\xi') + C_{RL}w_L(\xi')) d\xi' \right) (1 - w_R(\xi))
\]

\[
- \beta_1 \left( \int G_1(\xi - \xi')(w_R(\xi') + w_L(\xi')) d\xi' \right) w_R(\xi)
\]

LTP term

LTD term
Neurotrophic Factors

\[ N - (n_R + n_L) \xrightleftharpoons[\beta_2]{w_R} n_R \]

\[ N = \text{total amount of neurotrophic factor} \]
\[ n_R = \text{amount of neurotrophic factor taken up by right-eye afferents} \]

\[
\frac{dn_R}{dt}(\xi) = (N - (n_R(\xi) + n_L(\xi)))w_R(\xi) - \beta_2 n_R(\xi)
\]
Ocular Dominance Columns

V1 layer 4

LGN
left eye

LGN
right eye

$w_L$

$w_R$
Development of OD
Development of OD
Bifurcation Diagram: Control Case

\[ w_L \]

\[ \begin{array}{c}
N < N_m: \text{ Only one stable solution: } w_R = w_L = n_R = n_L = 0 \\
N_m < N < N_b: \text{ Two stable asymmetric steady-states } M_1 \text{ and } M_2 \text{ and one unstable steady-state } B \\
N = N_b: \text{ Saddle-node bifurcation} \\
N > N_b: \text{ Symmetric steady-state } B \text{ becomes stable } \Rightarrow \text{ a stable binocular solution}
\end{array} \]

\[ C_{RR} = 0.9 \]
\[ C_{LL} = 0.9 \]
\[ C_{RL} = 0.3 \]
\[ C_{LR} = 0.3 \]
Varying N: Control Case

N=0.1

N=2

N=4
Monocular Deprivation from Birth
Monocular Deprivation from Birth
Bifurcation Diagrams: Monocular Deprivation

\[ C_{RR} = 0.9 \quad C_{LR} = 0 \quad C_{RL} = 0 \quad C_{LL} = 0.6 \]

\[ \begin{align*}
N < N_b : \text{Two stable steady states } M_1 \text{ and } M_2 \\
N > N_b : \text{Second set of solutions appears via saddle-node bifurcation. The binocular solution is no longer symmetric}
\end{align*} \]
Varying N: MD from Birth

N=0.1

N=2

N=18
MD after Maturity
MD after Maturity
Interaction Function

\[ G(\xi - \xi') = A \cdot e^{-|\xi - \xi'|/\chi_1^2} - B \cdot e^{-|\xi - \xi'|/\chi_2^2} \]
Varying the Interaction Function

Less inhibition

MD from birth  OD  OD then MD

More inhibition

MD from birth  OD  OD then MD
Two layer model

V1 layer 2/3

$G_2(x - x')$

V1 layer 4

$G_1(\xi - \xi')$

$w_L(\xi)$

$w_R(\xi)$

left eye

LGN

right eye

LGN
Equations for Synaptic Strength in Second Layer

\[ \frac{dm}{dt}(x) = n_m(x) \left[ V_2(x) V_1(\xi) (1 - m(x)) \right] \]

LTP term

\[-\beta_{3} \left( \int G_2(x - x')m(x') dx' \right) m(x) \]

LTD term

\[ (1 - m) \quad \frac{K^+(n_m, V_2, V_1)}{m} \quad K^-(V_2) \]
Activity

\[ V_1(\xi) = \int G_1(\xi - \xi') \left( I_L w_L(\xi') + I_R w_R(\xi') \right) d\xi' \]

\[ V_2(\mathbf{x}) = \int G_2(\mathbf{x} - \mathbf{x}') m(\mathbf{x}') \int M(\mathbf{x}' - \xi) V_1(\xi) d\xi d\mathbf{x}' . \]
Amount of Neurotrophic Factor in Second Layer

\[ n_m(x) = \text{amount of neurotrophic factor at } x \]

\[ \frac{dn_m}{dt}(x) = (N - n_m(x))m(x) - \beta_4 n_m(x). \]
Ocular Dominance Columns in Two Layers
Normal OD Development
Delta Interaction Functions

Layer 4

Synaptic weights to layer 2/3

G2 is delta

G1 and G2 deltas
MD from Birth
Delta Interaction Functions

Layer 4
Synaptic weights to layer 2/3

G2 delta
G1 and G2 delta
Feedback

$G_2(x - x')$

$G_1(\xi - \xi')$

$\alpha_\varepsilon$

$w_L(\xi)$

$w_R(\xi)$

V1 layer 2/3

V1 layer 4

left eye

right eye

LGN

LGN
Equations for feedback

\[ \frac{dw_R}{dt}(\xi) = \]

\[ n_R(\xi) \cdot \int G_1(\xi - \xi') \left[ C_{RR}w_R(\xi') + C_{RL}w_L(\xi') + \alpha_\varepsilon m(x')(C_{RR}D_R(\xi') + C_{RL}D_L(\xi')) \right] d\xi' \]

- \beta_1 \left( \int G_1(\xi - \xi')(w_R(\xi') + w_L(\xi') + \alpha_\varepsilon m(x'))d\xi' \right) w_R(\xi)

\[ D_R(\xi') = \int M(\xi' - \xi'') w_R(\xi'') d\xi'' \]

contribution of right eye
Future directions

• Overcome computational obstacles to create ocular dominance map of layer 2/3

• Examine different time scales for the development of the different layers

• Consider non-constant feedback

• Explore different architectures for feedforward connections between layer 4 and layer 2/3
References

