Short Maturity Options and Jump Memory

We investigate "jump memory" using an extensive data base of short-term S&P 500 Index options. Jump memory refers to the attenuation of the implied jump intensity and magnitude parameters following a jump event. Behavioral and rational explanations for parameter attenuation are posited. A genetic algorithm is used to obtain implied parameter estimates. The pricing accuracy of the jump-diffusion model under nesting and parameter restrictions is also investigated. Nesting and parameter restrictions sharpens the remaining parameter estimates and has a negligible effect on pricing accuracy.

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INTRODUCTION

We use S&P 500 options on the spot and focus on the post-crash behavior of the jump component of option prices and implied jump-diffusion parameters. Under idealized jump-diffusion assumptions, the "crash event" produces an observation from the tail of the distribution. The price of the underlying falls and the option price adjusts accordingly. Under maintained assumptions, all implied parameters remain constant. However, there is extensive documentation in the literature that implied parameters change almost continuously, suggesting that the pricing model is incorrectly specified. Nevertheless, an investigation of the evolution of these parameters gives useful information on the behavior of market participants and can suggest improved model specifications. The signature finding of our investigation is that a type of jump memory apparently exists. Specifically, we note that large values of jump intensity and jump magnitude on the day of a crash event are followed by a period of exponential decline in these parameters. We posit competing behavioral and rational economic explanations for this phenomena.

Jump-diffusion models have proven to be useful in explaining option prices. The early work on option pricing models based on the jump-diffusion process is due to Merton (1976). His work is extended and/or tested further empirically in Ball and Torous (1985), Jorion (1988), Naik and Lee (1990), Bates (1991), Chang (1995), Bates (1996), Bakshi and Chen (1997), and others. Das and Sundaram (1999) provide evidence of jump diffusion models being able to generate sufficient skewness to price near term options where stochastic volatility models fail. Empirical testing by Bakshi, Cao, and Chen (1997) and Pan (2002) demonstrate the effectiveness of jump diffusion models relative to stochastic volatility models, generally finding a combination of the two being most effective. However, parameter estimates vary greatly and there is a high potential for stochastic volatility and jump components to substitute for each other. Eraker, Johannes and Polson (2002) test the two factor model developed by Duffie, Pan, and Singleton (2000) on S&P 500 and NASDAQ 100 daily returns. The Duffie, et.al., model generates jump-diffusions in both prices and volatility and accommodates volatility clustering, mean reversion, and correlations in the smooth diffusion innovation.

We use the jump-diffusion model as a vehicle for interpreting participant behavior. Our study extends the literature in several ways: First, we evaluate "jump memory" as it applies to the systematic attenuation of jump parameters following a market break. Second, we use a Genetic Algorithm as outlined by Dorsey and Mayer (1995) to obtain parameter estimates. The genetic algorithm is robust as we experience no "restarts" or failures in over 20,000 sets of parameter estimates. We also allow equal representation of out-of-the-money options in parameter estimates (Bakshi, Cao, and Chen's estimation is dominated by in-the-money options and Pan does not use out-of-the-money options). Third, we examine a parsimonious version of the four-parameter model. We set jump volatility to zero and alternately fix some combination of mean jump size (typically at -20% corresponding to the crash of 1987) or jump intensity. The remaining two free parameters are then implied. In an out-of-sample application, we find that the reduced version of the model is not dominated by the more complex four-parameter version of the model. Estimating only two-free parameters also allows us to find less noisy patterns of parameter attenuation following jumps.

The application in this paper is most closely related to that of Bates (1991) and Bates (2000). Bates (1991) uses S&P futures options to investigate the information content of out-of-the-money puts and implied jump-diffusion parameters prior to the crash of 1987. He finds evidence of downside risk in the year prior to the crash event but no strong fears in the two months leading up to the event. Bates uses data ending in 1987 and gave no detailed evidence of post-crash behavior. However, he notes that crash fear was present following the event. Bates (2000) examines post-crash fears and investigates the ability of alternative models to explain market prices. Although our dataset, model and parameter estimation techniques are different, we observe similar skewness phenomena.

Our data is an extensive database of short-term European options. Pricing these options is of interest in itself. A review of the finance literature reveals that options with less than ten days to maturity are largely ignored, e.g., see Bakshi, Cao and Chen (1997). The apparent reason for this neglect seems to be that the options market becomes structurally unstable for maturities of less than ten days. This instability combined with the linearity of the payoff function leads to difficulties in parameter estimation. For our sample of S&P 500 index options traded from January 2, 1987 through October 2, 1995, the market is dominated by shorter maturities (see Figure 1). Note that the peak volume is in contracts with *two* days to maturity and that contracts with less than 90 days to maturity form the overwhelming majority of trade volume. Table 1 is a further breakdown of these transactions by type (puts or calls) and moneyness. Several points are notable: 1) trading volume for puts and calls is dominated by the very short term, 2) trading volume in puts and calls does not differ significantly 3) put and call trading in expiring options is largely at-the-money (puts at 72% and calls at 77%) and 4) the majority of trades in calls in the longest maturity classes (64-180 and 183-386) are out-of-the money.

The flow of the paper is as follows: Section 1 briefly reviews the jump-diffusion option pricing model. The empirical pricing analysis is developed in Section 2. The performance of the jump diffusion model under nesting and parameter restrictions is also developed in this section. Section 3 focuses on the post-crash behavior of parameter and moment estimates. Section 4 concludes.

1. THE JUMP-DIFFUSION MODEL

Merton (1976) assumes that jump risk is diversifiable, thus permitting returns on the otherwise risk-free replicating portfolio to be equated to the risk-free rate as a means of deriving the fundamental partial differential equation. Chang (1995) determines that Merton's result is consistent with no-arbitrage only when the pricing standard (stochastic discount factor) does not contain a jump component. Chang questions the economic intuition of a model that does not allow for jumps in consumption.

We use Bates' (1991) setup and rationale for the jump-diffusion model. Bates notes that it is implausible to assume that returns on the S&P 500 index are unsystematic. However, by imposing restrictions on tastes and preferences, he derives an appropriate risk-neutral jump-diffusion that is identical to the original jump-diffusion in distributional characteristics. In general, parameters of this risk-adjusted process are different from the original except for diffusion volatility and the instantaneous cost-of-carry in the drift term. However, since the parameters of the process are inferred by option prices, risk preferences and tastes are imbedded in the implied parameters. Accordingly, we do not further distinguish between these processes and write the risk-neutralized jump-diffusion as :

$$\frac{dS}{S}=(b-\lambda\bar{k})dt+\sigma dB+kdq, \tag{1}$$

where dB is standard Brownian Motion, $b=r-d$ is the asset's cost-of-carry and σ is the diffusion volatility. Other parameters relating to the jump component are λ, the Poisson intensity, and $\bar{k}\equiv E[k]$, the expected proportional jump size. The random increment dq corresponds to the presence or absence of a jump. A jump occurs with probability $\lambda dt+o(dt)$ and corresponds to $dq=1$. Otherwise $dq=0$. The random jump magnitude is k and $ln\,(1+k)\sim N(\gamma-.5\delta^2,\delta^2)$, giving $E\,(k)=e^{\gamma}-1$.

The appeal of the jump-diffusion process relative to a pure diffusion stems in part from its ability to generate appropriate volatility, skewness and kurtosis measures. These measures for $\ln\left(\frac{S_T}{S_0}\right)$ can be computed as follows:[1]

$$Volatility=\sigma_{JD}=\sqrt{\lambda\left(\gamma'^2+\delta^2\right)+\sigma_0^2}, \tag{2}$$

$$Skewness=\frac{\lambda\gamma'(\gamma'^2+3\delta^2)}{(\sigma_{JD})^3\sqrt{T}}, \tag{3}$$

$$Kurtosis=3+\frac{\lambda(\gamma'^4+6\gamma'^2\delta^2+3\delta^4)}{T(\sigma_{JD})^4}, \tag{4}$$

where $\gamma'=\gamma-\frac{\delta^2}{2}$. Equations 3 and 4 underscore the effect of maturity on the behavior of skewness and kurtosis. Both measures explode as $T\rightarrow 0$ while approaching that of a normal distribution as maturity increases. More specifically, skewness$\rightarrow O\left(\frac{1}{\sqrt{T}}\right)$ and kurtosis$\rightarrow 3+O\left(\frac{1}{T}\right)$.

Under standard assumptions, an instantaneous interest rate r and dividends d, calls and puts at strike price X with time to maturity T are priced according to:

$$C(S,T)=e^{-rT}\sum_{n=0}^{\infty}\frac{e^{-\lambda T}\left(\lambda T\right)^n}{n!}\left(Se^{b(n)T}N(d_{1n})-XN(d_{2n})\right), \tag{5}$$

[1] These measures must be regarded as characteristics of the risk-neutral distribution when they are computed from risk-neutral parameters.

$$P(S,T)= e^{-rT} \sum_{n=0}^{\infty} \frac{e^{-\lambda T} (\lambda T)^n}{n!} \left(XN(-d_{2n}) - Se^{b(n)T} N(-d_{1n}) \right), \tag{6}$$

where $b(n) = r - d - \lambda\bar{k} + \frac{n\gamma}{T}$, $N(y)$ is the cumulative standard normal distribution evaluated at y, and

$$d_{1n}=\frac{\ln(\frac{S}{X}) + b(n)T + \frac{1}{2}(\sigma^2 T + n\delta^2)}{\sqrt{\sigma^2 T + n\delta^2}},\ d_{2n} = d_{1n} - \sqrt{\sigma^2 T + n\delta^2}. \tag{7}$$

2. EMPIRICAL ANALYSIS OF S&P 500 OPTIONS

We analyze several aspects of the S&P 500 option on the spot. A genetic algorithm and the least squares merit function are used to imply the parameters of the jump-diffusion process. Nested versions of the jump-diffusion model are also parameterized and evaluated by pre-specifying one or more of the four parameters. This approach is in contrast with more highly parameterized models that allow for jumps in both the underlying and volatility state variables. These models provide some improvement in fit at the margin, but a flat optimum in the error surface leads to unstable parameter estimates. Because we wish to interpret parametric evolution over time, we choose the simpler jump-diffusion model with jumps only in the underlying. In short, the benefits of reduced dimensionality outweigh the negligible loss of pricing accuracy.

2.1. Data

The data consists of option prices, the price of the underlying and treasury bill quotes. The option is the Standard and Poors' 500 Index (SPX) put option that trades on the Chicago Board of Exchange (CBOE). These widely traded European options are used for a variety of purposes, including risk management, asset allocation and investing. Beginning on the third Friday of each month, we record tick-data on the two-month put option price on a daily basis until the option expires. In a given year, there are twelve daily data series of this nature. The quotes we use begin in January 2 of 1987 and extend through October 2, 1995.

For each day, closing treasury bill quotes that straddle a given option's maturity date are collected from the *Wall Street Journal*. From the quote midpoint, a risk-free rate for the option is calculated as the time-weighted average of the two quotes straddling the option maturity date.

The present value of the anticipated index dividend payout is calculated using put-call parity. This calculation is updated every fifteen minutes throughout the day so as not to become stale when it is applied in the data collection process.

To generate a data series, put option quotes are separated into twelve moneyness classes. A moneyness class is determined by the ratio of the dividend adjusted index level to the present value of the strike price. To allow representation of all strikes, options are classified into 12 different moneyness ratios ranging from below 0.85 to above 1.15 with 10 equally spaced intervals in between. Only one option quote is used to represent each class. At 12:00 noon CST, the outstanding put option quotes for the twelve moneyness classes are sampled. The noon sample is used to calibrate the option-pricing model (using quote midpoints). Noon quotes on the following day are out-of-sample and are used to test the model. Market prices for puts are calculated as the average of bid and ask prices.

2.2. Parameter Estimation

Jump-diffusion parameters are estimated daily by a genetic algorithm and compared with those obtained by a gradient search technique. The criterion function is the minimum sum of squared errors between model prices and market prices,

$$\min_{(\sigma\gamma\lambda\delta)} L = \sum_{i=1}^{n} \left[P_{Model_i} - P_{Market_i}\right]^2, \tag{8}$$

where n is the number of moneyness classes. The observations are the midpoint of the quotes at noon for up to 12 different strikes ($n \leq 12$). Observations on at least 5 moneyness classes were required for inclusion of that day in the sample.[2]

Genetic procedures have begun to appear recently in the finance literature. To date, these codes have been used primarily to investigate trading rules as in Neely, Weller and Dittmar (1997) or Allen and Karjalainen (1999). The procedure used here follows the approach of Dorsey and Mayer (1995). Their algorithm is initiated by selecting a range for each of the variables in the parameter search as opposed to the initial guess vector found in gradient search routines. The genetic code randomly searches the ranges allowing promising solutions a high

[2] Following Hull (1999), we used trading days for volatility calculations and calendar days for discounting.

probability of remaining in the population of candidate solutions passed to the next generation. New solutions are based on the desirable traits found in previous generations. Undesirable traits are passed on to the next generation with lower probability and should disappear after a number of generations. The genetic algorithm typically compares favorably with gradient search techniques when there are multiple optima. Specifically, the algorithm is not "trapped" by local optima. Instead, the algorithm continues to seek global optimal solutions according to predefined genetic rules. Conversely, gradient search techniques depend on the starting guess and can frequently converge to local optima when applied to a complex surface.

Table 2 gives the average estimates and their sample standard deviation for all four jump diffusion parameters and for volatility, skewness and kurtosis. The statistics are based on 3665 daily estimates from January 7, 1987 through October 2, 1995 and are further segmented by maturity class. In every case, the estimates have greater standard deviation for the short term maturities, as expected. For example, the standard deviation of implied volatility is more than five times greater for the 0-12 day maturity class than for the 43-61 day class. Mean estimates also increase for expiring options. The diffusion annualized sigma (σ_0) is most stable over maturity classes and averages about 11% with a standard deviation of about 4.25%. Total volatility increases from 0.2654 for the longest maturity class to 1.9146 to the shortest maturity class. Skewness for the shortest maturity class averages -0.6785 and -0.5972 for the longest maturity class. The corresponding numbers for kurtosis are 4.7514 and 3.8026, underscoring the increasing departures from implied normality for short maturities.

The stability of jump-diffusion parameters is depicted graphically in Figure 2. As noted above, diffusion volatility is most stable at around 11% while we observe drastic increases in average values and in the stability in δ and λ at maturities less than 10 days. The average jump size, γ, is negative and increases slightly in magnitude (decreases algebraically) at shorter maturities. It appears as if bets on increasing volatility and jump intensity become more important in this maturity range. A plausible explanation for this result is a clientele of very short term participants with different subjective risk assessments. Figure 3 is a plot of total volatility, skewness and kurtosis versus days-to-maturity. Again, we note the increasing magnitude of these measures becoming pronounced at maturities of less than 12 days. Skewness and kurtosis measure are also volatile for maturities less than 12 days.

2.2.1. Restricted Parameter Estimation

Estimating all four parameters by the genetic algorithm is time consuming. In a ten day sample, the average daily time to convergence is 10.70 minutes. When two parameters are fixed, the average daily time to convergence is 4.38 minutes. Algorithm convergence is defined as an improvement in sum of squared errors less than 0.0001. All estimates are done in compiled FORTRAN using an AMD Athlon 2100+ processor with 512 MB of RAM.

A case can be made in favor of the restricted parameter model if there is negligible loss in pricing accuracy and/or advantages in interpretation. Five models are evaluated. They are defined as:

Model	1	2	3	4	5
Parameter					
σ	B-S	B-S	estimated	estimated	estimated
γ	0	estimated	estimated	-0.2	estimated
λ	0	0.2	0.2	estimated	estimated
δ	0	0	0	0	estimated

Model 1 is the standard Black-Scholes Model. In Model 2, Black-Scholes implied volatility is used in combination with estimates of γ and fixed choices for λ and δ. Models 3 and 4 use estimates of σ and either γ (Model 3) or λ (Model 4). Models 1 through 4 have $\delta = 0$ and are therefore nested in the full parameter model (Model 5). Since $\delta = 0$, all jumps in in the nested models are of magnitude γ.

Choices of λ and γ are intuitive but somewhat arbitrary. They were not chosen to agree with the averages of estimates obtained in the unrestricted model since this assumes access to out-of-sample data. Instead, we choose an average time between market "breaks" of five years

($\lambda = 0.20$) and a magnitude of 20% ($\gamma = -0.20$). The actual estimates using the full dataset are given in Table 2.

Table 3 gives the average in-sample pricing errors (model - market) for each model by moneyness and maturity. Parameters are estimated at noon CST, and subsequently tested on these same data. When averaged over all moneyness levels, Models 3, 4 and 5 are almost indistinguishable. Their average errors are \$0.015, -\$0.036 and -\$0.013, respectively. The Black-Scholes Model (Model 1) and Black-Scholes variant (Model 2) had average errors of -\$0.199 and \$0.105. Because these are puts, high moneyness values (index/strike) are out-of-the money. However, we noted no apparent systematic errors across moneyness and maturities for Models 3, 4 and 5.

Table 4 gives average *out-of sample* errors for all models by moneyness and maturity. In this experiment, parameters are estimated daily and model errors are computed on the following day. Models 4, 5 and 6 also perform better here and, in fact, the restricted Model 4 has average error of \$0.043 compared to \$0.064 for the unrestricted model. No striking systematic patterns are apparent except the underpricing of far-in-the money puts for the shortest two maturity classes (1-12 days and 15-26 days)

3. JUMP MEMORY

Parameter shifts have been studied extensively in the option pricing literature. Bates (1991), in particular, looked at changes in implied parameters and their information content. Shifts in the parameters prior to significant events are theoretically important because information leakage calls into question the efficiency of the market. Instead of pre-event behavior, we focus on post-event behavior and find that the existence of an apparent "crash memory" is reflected in jump components of option price moments and in the (risk-neutralized) implied parameters. Specifically, this means that implied parameters reach unprecedented levels during the event and then gradually attenuate over time. This too has ramifications with respect to market efficiency if it can be established that market participants systematically revise their subjective forecasts following market breaks using only the recency of the jump in price as a basis for these revisions. There is some behavioral support for this notion. Baddeley (1986), for example, observes the recency effect in both short and long-term memory experiments.

The crash events are referred to as the crash of 87, the mini-crash of 89 and the Gulf War. The crash of 87 was on 10/19/87. On that day, the S&P 500 fell 20.4% from its 10/16 close. The mini-crash of 89 was on 10/13/89. On that day the S&P dropped 6.1% from its 10/12 close. The Gulf War began at 7 pm EST on 1/16/91. While the outcome was more or less apparent by 2/1/91, the actual suspension of military combat was not ordered by President Bush until 2/27/91. The pre- and post-periods are defined as -270 and +90 days for both crashes and the Gulf War. The post-crash period for the Gulf War began on 2/27/91.

3.1. Implied Time Series

Restricted forms of the jump-diffusion model reduce dimensionality and give an interesting picture of the jump memory phenomena. We first examine four time series: the percent of option price explained by jumps, implied jump size (γ), implied jump intensity(λ),and implied diffusion (σ_0). These series are shown graphically in Figures 4, 5 and 6 and further categorized by crash events in Table 5.

The jump percentage series is depicted in Figure 4. We derive this series by decomposing jump-diffusion put option prices according to

$$P(S,T)=\sum_{n=0}^{\infty} P_r(n)P_n, \tag{9}$$

where P_n is the contribution to option price conditional on n jumps. Equation (9) allows us to characterize the percent contribution to put option prices from the n^{th} jump as $\frac{P_r(n)P_n}{P(S,T)} \times 100\%$. Close-to-the money options ($0.99 < s/x < 1.01$) with more than 28 days to maturity are priced using Model 3. These screens are chosen to mitigate the effects of maturity and moneyness, leaving 1714 observations. Data from Table 5 and Figure 4 indicate that in the 270 days prior to the 10/19/87 crash, an average of about 11.08% of option price is explained by jumps, increasing to about 95% at the time of the crash. In the 90 days following the crash the corresponding average is 74.24%. After this, the percent diminishes in an exponential fashion to about 30% in just over 1.5 years. The same phenomena is observed in the mini-crash of 89 . In the 270 days preceding the crash, an average of 29.89% of option price is explained by jumps, increasing to an average of 61.83% in the 90 days following the crash. There are also permanent crash effects. While the pre-crash average is about 11%, this figure stabilizes to about 35% in the four years following the Gulf War.

The implied σ and λ time series are shown in Figure 5. We observe extreme values of λ and σ in October of 1987, October of 1989 and January of 1991. While σ is a single spike on these dates, λ attenuates to steady state after slightly more than one year following the 87 and 89 crashes. Since λdt is the local probability of a jump, these results show that market participants first spike up jump intensity then systematically reduce these probabilities following major crash events. Table 5 provides more precise information. Jump intensity averaged 0.1494 pre-crash 87, 2.367 post-crash period and leveled off to 0.2855 in the final four years of our data. These estimates translate into implied crashes every 6.68 years, 0.42 years and 3.5 years, respectively.

The implied σ and γ time series are shown in Figure 6. The behavior of σ is similar to that observed in Figure 5, i.e., there are spikes in σ but no significant attenuation effects. Also note the large post-crash event changes in γ and the subsequent attenuation. Table 5 gives pre- and post-crash values for γ of -0.0908.and -1.6371 for the crash of 87. The corresponding pre- and post-values for the mini-crash of 89 are -0.2209 and -0.7209. As with jumps, we also note a permanent change in γ. While the pre-crash average value is about -9%, this figure stabilized to approximately -20% in the four years after the Gulf War.

The Gulf-War effect on these series is dissimilar to the effects of the crashes of 87 and 89. Note specifically in Table 6 and Figures 4,5, and 6 that the percent jump series and the implied parameters series reflect increasing uncertainty pre-Gulf War. However, these series immediately return to near normal levels following the Gulf War. Thus, post-event data suggests that the crashes of 87 and 89 introduced uncertainty while the Gulf War reduced uncertainty. Similarly, pre-event data is consistent with the interpretation that the crashes of 87 and 89 were surprises while pre-Gulf War tensions increased over time.

3.2. Volatility, Skewness and Kurtosis

Total volatility, skewness and kurtosis exhibit the same general pattern as observed for the implied parameters. However, the attenuation effect is not pronounced except in the volatility time series following the crash of 87.[3] Panels A, B and C in Table 6 display pre- and post-event moments for Model 5 (unrestricted parameters), Model 3 and Model 4, respectively. Data in these panels show that the implied moments tend to be more stable for Models 3 and 4 than for Model 5. For example, in the crash of 87, Model 5 (Model 4) pre-event volatility increases from 30.19 (19.66)% to 258.88 (42.34)% post-event. In the mini-crash of 89, Model 5 (Model 4) pre-event volatility increases from 20.32 (15.88)% to 24.27 (21.14)% and in the Gulf War, Model 5 (Model 4) pre-event volatility decreases from 40.36 (23.09)% to 32.56 (17.16)% post-event. The final period volatility of 73.89% (based on 1863 observations) implied by the unrestricted model underscores the problem. This result is in large part due to several observations with an extremely high values of δ. When restricted Models 3 and 4 are used, the same dataset produces volatilities of 15.25% and 14.36%, respectively.

4. DISCUSSION

On the 87 and 89 crash dates, we observe large changes in implied volatility, the intensity of jumps, jump magnitude and jumps as a percent of price. We also observe that these series tend to systematically attenuate over a period of about 1.5 years. We refer to this systematic attenuation as jump memory. The field of "Behavioral Economics" potentially offers some insights on these observations.

Behavioral economics replaces strong rationality assumptions used in economic modeling with assumptions that are based on findings in psychology. For a survey of the Behavioral Economics approach, see Camerer (1995), Rabin (2001) and Shefrin (2002). In the main, these theories are not normative but instead are attempts to explain or predict the micro-behavior of market participants. They generally take expected utility theory or Bayesian updating as a point of departure and try to explain deviations from the theory by cognitive mechanisms or hedonic

[3]Figures for the volatility, skewness and kurtosis series are available from the authors on request.

sensations. For example, Kahneman and Tversky (1979) find that losses are disliked about twice as much as equal sized gains. In the present context, these theories suggest that crash fear is overweighed because of the loss factor and the recency effect. The loss factor might explain implied parameter jumps while the recency factor might explain their observed attenuation over time. Parameter attenuation which proxies for "crash-fear" is also consistent with the overreaction hypotheses of DeBondt and Thaler (1985, 1990).

Under some scenarios, puts can be mispriced as a result of crash fear. Suppose, for example, that asset prices on the event date are draws from the lower tail of a jump-diffusion probability density. Furthermore, suppose the density function has constant parameters. The draw from the lower tail could be the result of an improbable sequence of bits of information unfavorable to asset price. Because of crash fear, investors may interpret this observation as indicative of a change in parameters. Thus, they systematically revise their subjective parameters and overvalue put options. Even if parameters change in the posterior distribution, the overweighing of loss events might result in overvalued puts. As crash fear attenuates, parameters return to approximate pre-event levels.

There are also rational economic explanations. Puts can be priced correctly if crash fear and implied parameter revisions are due to fundamental factors that suggest increased price uncertainty. For example, international banking crises may raise the spectre of defaults, contagion, increased regulation and trade frictions. If subsequent information serially resolves the increased uncertainty, parameters which proxy for risk may attenuate in a systematic fashion. But this is consistent with the rational revision of distributions as new information is revealed. The result is that puts are rationally priced. The stylized facts of the crash of 87 and the mini-crash of 89 are not inconsistent with such behavior.[4] Furthermore, the stylized facts of the Gulf War support the immediate and rational revision of uncertainty by investors. The prior distribution reflected a sustained period of increased tensions. However, the posterior distribution did not attenuate but immediately returned to normal (non-war) levels.

[4] In fact, the crash of 87 has been attributed in part to a failure in liquidity. The result is that option pricing models were temporarily invalidated since dynamic hedging strategies were inoperable.

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Table 1. Transaction volume by moneyness and maturity. 1987-1995

DTM	Volume	Percent	Moneyness (%)		
			In	At	Out
Panel A: Calls			>1.03		<0.97
0-12	5,712,374	11.40	8.94	76.89	14.17
15-26	5,016,280	10.01	5.86	66.01	28.14
29-40	4,108,545	8.20	7.17	57.83	35.00
43-61	3,083,277	6.15	3.94	45.52	50.54
64-180	4,129,514	8.24	7.85	37.88	54.27
183-386	603,200	1.20	11.14	24.00	64.68
Panel B: Puts			<0.97		>1.03
0-12	6,164,571	12.30	2.40	71.68	25.92
15-26	6,151,639	12.27	3.10	58.68	38.22
29-40	4,715,987	9.41	2.79	50.72	46.49
43-61	3,354,180	6.69	5.74	41.07	53.18
64-180	5,338,367	10.65	6.87	30.88	62.25
183-386	1,747,575	3.49	9.20	21.77	69.03
Panel C:	Calls+Puts	Calls	Puts	Calls%	Puts%
0-12	11,876,945	5,712,374	6,164,571	48.10	51.90
15-26	11,167,919	5,016,280	6,151,639	44.92	55.08
29-40	8,824,532	4,108,545	4,715,987	46.56	53.44
43-61	6,437,457	3,083,277	3,354,180	47.90	52.10
64-180	9.467,881	4,129,514	5,338,367	43.62	56.38
183-386	2,350,775	603,200	1,747,575	25.66	74.34

DTM is days-to-maturity. In-, at- and out-of-the-money for puts is defined by $m < 0.97$, $0.97 \leq m < 1.03$ and $1.03 \leq m$, respectively, where $m = \textit{index price/strike price}$.

Table 2. Parameter estimates.

Maturity	σ_0	λ	γ	δ	$\gamma^{'}$	Vol	Skew*	Kurt*
Means								
0-12	0.1103	1.8434	-0.2111	1.1371	-2.7492	1.9146	-0.6785	4.7514
15-26	0.1117	1.9628	-0.1255	0.4559	-0.9926	0.6180	-0.4578	3.8784
29-40	0.1089	1.7827	-0.1109	0.2053	-0.4855	0.3985	-0.5199	3.7367
43-61	0.1047	1.6779	-0.1223	0.1318	-0.3478	0.2654	-0.5973	3.8026
Std Dev								
0-12	0.0577	3.6910	0.3399	1.9470	9.5855	5.2481	0.5298	2.3989
15-26	0.0408	2.6963	0.1765	1.2362	3.8790	1.5802	0.4082	1.3565
29-40	0.0402	2.0630	0.1311	0.8412	3.1041	2.0606	0.3540	1.2722
43-61	0.0383	1.9528	0.1334	0.6587	3.8872	1.0065	0.3088	0.9098
All								
Means	0.1083	1.7973	-0.1322	0.3679	-0.8703	0.6132	-0.5541	3.9336
Std Dev	0.0426	2.4735	0.1878	1.1582	4.9782	2.4865	0.3889	1.4391

Volatility, skewness and kurtosis of $\ln(S_T/S_0)$, annualized. Based on 3665 estimates from 1/7/87 through 10/2/95.

Table 3. In-sample pricing errors.

Model	1	2	3	4	5	1	2	3	4	5
Parameter										
σ	B-S	B-S	est	est	est	B-S	B-S	est	est	est
γ	0	est	est	-0.2	est	0	est	est	-0.2	est
λ	0	0.2	0.2	est	est	0	0.2	0.2	est	est
δ	0	0	0	0	est	0	0	0	0	est
	Average errors: model - market									
Moneyness	DTM=1-12					DTM=15-26				
≤ 0.94	-0.311	-0.149	-0.137	-0.325	-0.058	-0.246	-0.153	-0.202	-0.298	-0.099
≤ 0.97	-0.090	0.104	0.030	-0.170	0.070	0.244	0.362	0.100	0.029	0.091
≤ 1.03	-0.004	0.204	0.006	0.003	-0.005	0.038	0.306	-0.017	0.007	-0.012
≤ 1.06	-0.441	-0.164	-0.094	0.011	0.034	-0.549	-0.193	-0.101	-0.028	0.015
> 1.06	-0.532	-0.142	0.014	0.061	0.039	-0.605	-0.201	0.092	0.100	0.024
All Money	-0.205	0.000	-0.054	-0.146	-0.007	-0.191	0.043	-0.045	-0.061	-0.011
Moneyness	DTM=29-40					DTM=43-61				
≤ 0.94	-0.015	0.019	-0.113	-0.237	-0.118	0.374	0.488	0.031	-0.116	-0.081
≤ 0.97	0.560	0.731	0.207	0.147	0.088	0.795	1.019	0.211	0.136	0.050
≤ 1.03	0.040	0.425	-0.062	-0.033	-0.021	0.007	0.443	-0.127	-0.072	-0.023
≤ 1.06	-0.742	-0.258	-0.173	-0.066	0.008	-0.909	-0.378	-0.186	-0.060	0.020
> 1.06	-0.733	-0.309	0.103	0.102	0.001	-0.899	-0.463	0.136	0.119	-0.013
All Money	-0.208	0.121	-0.008	-0.018	-0.015	-0.194	0.167	0.010	0.001	-0.015
All Options	-0.199	0.105	-0.015	-0.036	-0.013					

DTM is days to maturity. Est means parameters were estimated. Otherwise parameters were fixed for the entire testing period as indicated. Cell entries are (model price-market price). 3735 observations.

Table 4. Out-of-sample pricing errors. One day after parameterization

Model	1	2	3	4	5	1	2	3	4	5
Parameter										
σ	B-S	B-S	est	est	est	B-S	B-S	est	est	est
γ	0	est	est	-0.2	est	0	est	est	-0.2	est
λ	0	0.2	0.2	est	est	0	0.2	0.2	est	est
δ	0	0	0	0	est	0	0	0	0	est
	Average errors: model - market									
Moneyness	DTM=1-12					DTM=15-26				
≤ 0.94	-0.343	-0.197	-0.192	-0.361	-0.109	-0.268	-0.199	-0.253	-0.329	-0.155
≤ 0.97	-0.111	0.079	0.017	-0.179	0.063	0.296	0.404	0.150	0.077	0.133
≤ 1.03	0.269	0.478	0.272	0.273	0.246	0.197	0.473	0.135	0.147	0.126
≤ 1.06	-0.328	-0.032	0.038	0.151	0.175	-0.466	-0.107	-0.014	0.067	0.120
> 1.06	-0.483	-0.076	0.074	0.151	0.120	-0.565	-0.141	0.178	0.180	0.108
All Money	-0.111	0.086	0.023	-0.061	0.068	-0.123	0.107	0.018	0.002	0.047
Moneyness	DTM=29-40					DTM=43-61				
≤ 0.94	-0.014	0.198	-0.028	-0.187	-0.042	0.329	0.436	-0.009	-0.154	-0.139
≤ 0.97	0.625	0.083	0.329	0.249	0.216	0.852	1.083	0.283	0.207	0.106
≤ 1.03	0.207	0.640	0.138	0.162	0.167	0.116	0.567	-0.023	0.034	0.091
≤ 1.06	-0.629	-0.130	-0.048	0.075	0.150	-0.844	-0.321	-0.123	0.011	0.100
> 1.06	-0.695	-0.253	0.197	0.200	0.087	-0.868	-0.435	0.200	0.182	0.042
All Money	-0.125	0.246	0.120	0.102	0.109	-0.141	0.217	0.067	0.057	0.035
All Options	-0.129	0.185	0.066	0.043	0.064					

DTM is days to maturity. Est means parameters were estimated. Otherwise parameters were fixed for the entire testing period as indicated. Cell entries are (model price-market price). 3735 observations.

Table 5. Implied parameters

Crash Event	Start Date	End Date	Jumps %	Average γ	Average λ	# Obs γ, λ	Jumps
Pre- 87	1/22/87	10/19/87	11.08	-0.0908	0.1494	269	134
Post- 87	10/19/87	1/17/88	74.24	-1.6371	2.3666	105	52
Pre- 89	1/16/89	10/13/89	29.89	-0.2209	0.2475	251	128
Post- 89	10/13/89	1/11/90	61.83	-0.7209	0.7386	99	40
Pre-Gulf	4/21/90	1/16/91	59.55	-0.4927	0.8408	347	122
Post-Gulf	2/27/91	5/28/92	28.28	-0.2286	0.2708	102	54
Final	5/28/91	10/2/95	34.90	-0.1899	0.2855	1853	912

The Jumps column is the percent of price explained by one or more jumps. To compute this column, parameters from Model 3 were used to price components of at-the-money options (days-to-maturity>28 and moneyness between 0.99 and 1.01). The parameters γ and λ are average sse estimates using Model 3 and Model 4, respectively. The number of observations for the parameter estimates and at- the-money options are given in the last two columns. The last row (Final) refers to the values in the last part of our dataset

Table 6. Volatility, skewness and kurtosis

Crash Event	Start Date	End Date	*Average Vol%*	*Average Skew*	*Average Kurtosis*	Number Obs
Panel A: Model 5: Unrestricted model						
Pre- 87	1/22/87	10/19/87	30.19	-0.1895	3.6214	269
Post- 87	10/19/87	1/17/88	258.88	-0.6106	4.4240	105
Pre- 89	1/16/89	10/13/89	20.32	-0.4684	3.7664	251
Post- 89	10/13/89	1/11/90	24.27	-0.7613	4.1891	99
Pre-Gulf	4/21/90	1/16/91	43.46	-0.6308	3.8667	347
Post-Gulf	2/27/91	5/28/92	32.56	-0.4728	3.8772	102
Final	5/28/91	10/2/95	73.89	-0.5848	4.0305	1853
Panel B: Model 3: $\lambda = 0.20$, $\delta = 0$.						
Pre- 87	1/22/87	10/19/87	20.31	-0.0996	3.2802	269
Post- 87	10/19/87	1/17/88	102.96	-0.8430	6.3436	105
Pre- 89	1/16/89	10/13/89	16.23	-0.5616	3.8388	251
Post- 89	10/13/89	1/11/90	37.62	-1.2998	5.6655	99
Pre-Gulf	4/21/90	1/16/91	31.26	-0.9333	5.0029	347
Post-Gulf	2/27/91	5/28/92	18.17	-0.4935	3.8228	102
Final	5/28/91	10/2/95	15.25	-0.6350	4.4478	1853
Panel C: Model 4: $\gamma = -0.2, \delta = 0$.						
Pre- 87	1/22/87	10/19/87	19.66	-0.1635	3.1742	269
Post- 87	10/19/87	1/17/88	42.34	-0.2638	3.1402	105
Pre- 89	1/16/89	10/13/89	15.88	-0.4918	3.6242	251
Post- 89	10/13/89	1/11/90	21.14	-0.5946	3.5840	99
Pre-Gulf	4/21/90	1/16/91	23.09	-0.4919	3.4564	347
Post-Gulf	2/27/91	5/28/92	17.16	-0.4148	3.4868	102
Final	5/28/91	10/2/95	14.36	-0.7208	4.0544	1853

The last row (Final) refers to the average values for the four years following the Gulf War. Risk-neural skewness and kurtosis computed from equations (3) and (3)

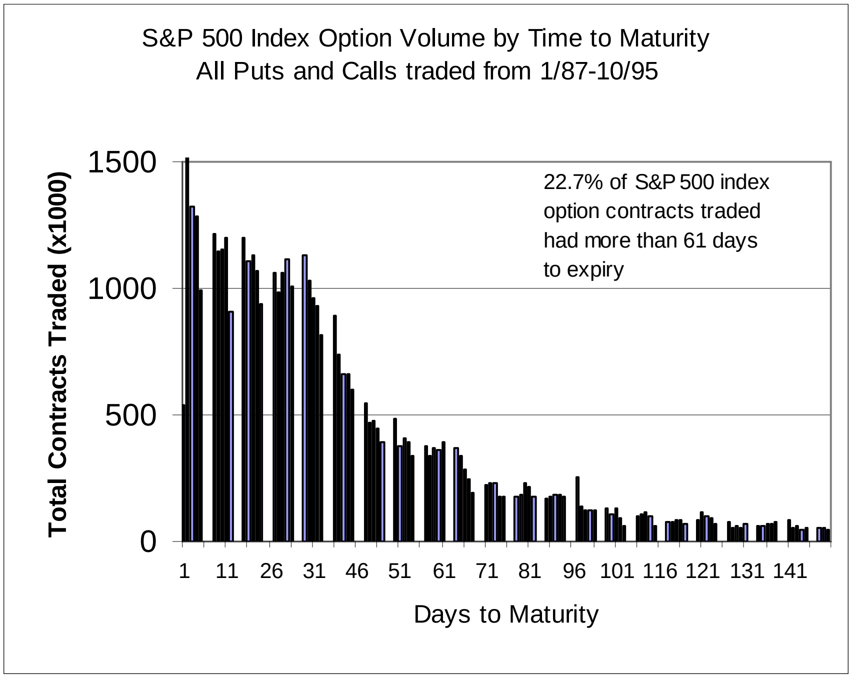

FIG. 1. Volume of trades: 01/02/87 to 10/02/95

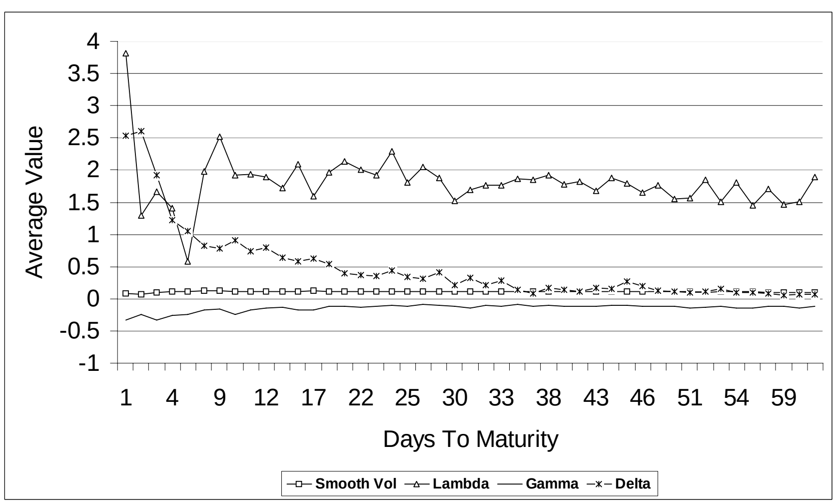

FIG. 2. Jump diffusion parameters and days to maturity.

Based on 3665 observations from 01/07/87 through 10/02/95.

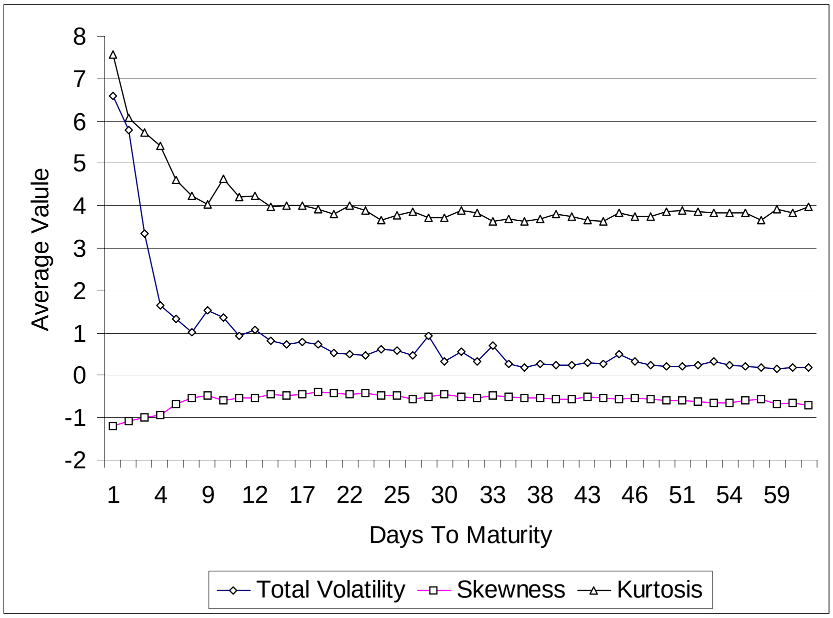

FIG. 3. Volatility, skewness and kurtosis of $\ln(S_T/S_0)$

Based on 3665 observations from 01/07/87 through 10/02/95.

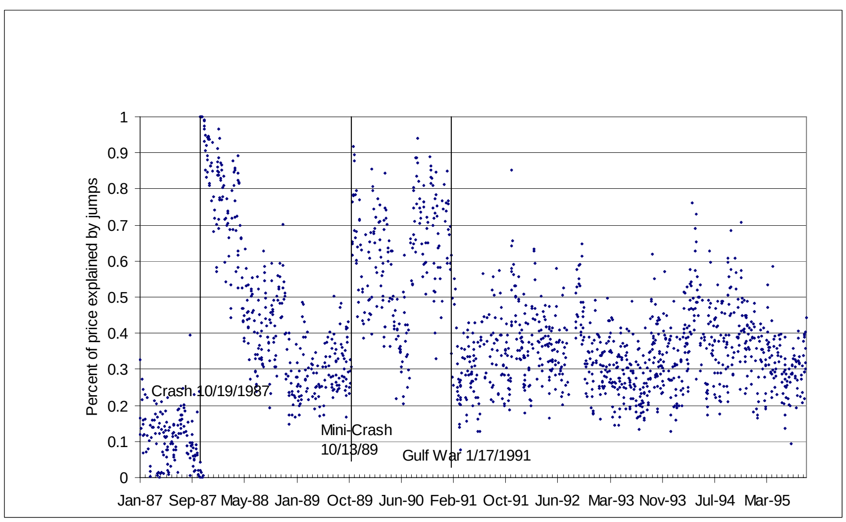

FIG. 4. Percent of price explained by jumps

Lambda = 0.2, delta =0. Other parameters are estimated using minimum sum of squares and the genetic algorithm. In-sample data from 01/07/87 through 10/02/95.

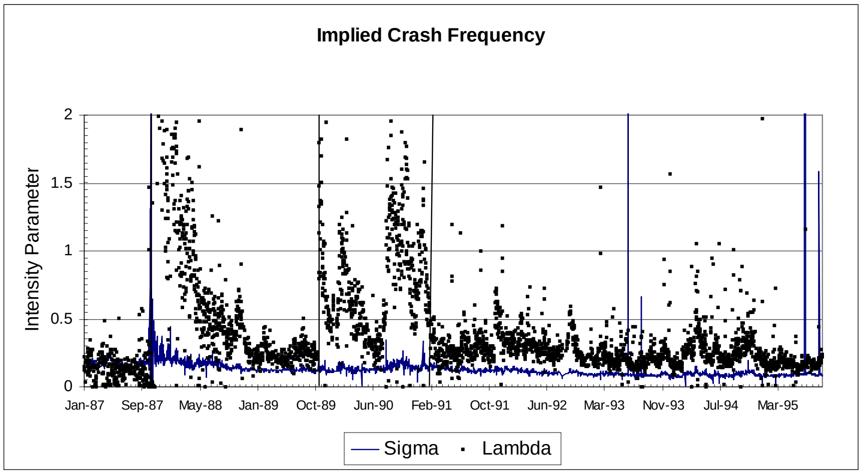

FIG. 5. Implied σ and λ

Gamma =-0.2, delta = 0. Other parameters are estimated using minimum sum of squares and the genetic algorithm. In-sample data from 01/07/87 through 10/02/95.

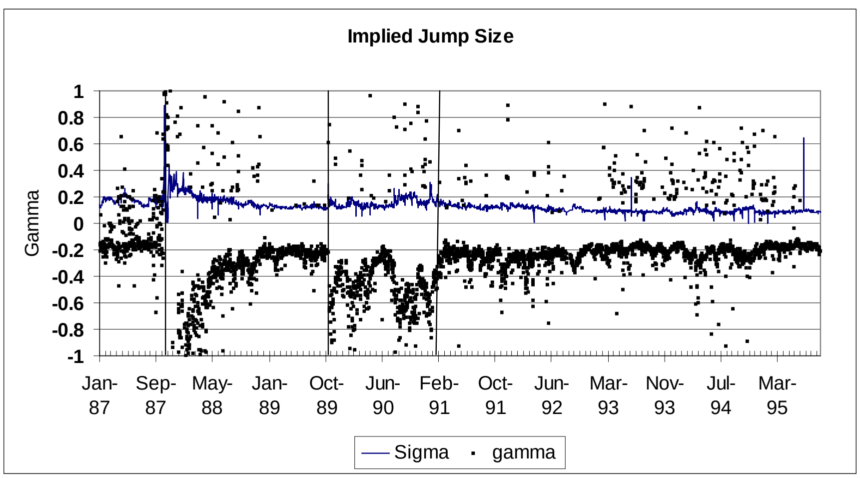

FIG. 6. Implied σ and γ.

Lambda = 0.2, delta = 0. Other parameters are estimated using minimum sum of squares and the genetic algorithm. In-sample data from 01/07/87 through 10/02/95.