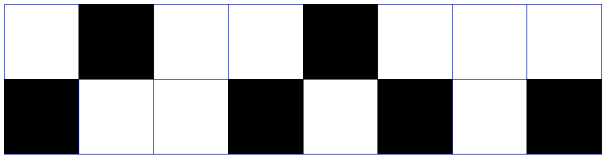
Adjacency-free selections of squares from a 2×N grid

Here we have a selection of 6 squares from a (2x8) grid, with no two squares in the selection (horizontally or vertically) adjacent:



It is not hard to derive the following formula for the number **Af[n,k]** of adjacency-free selections of *k* squares from a (2×*N*) grid:

```
In[1]:= Af[n_, 0 _] := 1;
        Af[n_, k_] := Sum[2^r Binomial[k - 1, r - 1] Binomial[n - k + 1, r], {r, 1, k}]
```

We can, for instance, make the following table in the style of Pascal's Triangle:

```
In[3]:= AfTriangle = Table[Af[n, k], {n, 0, 12}, {k, 0, n}];
        AfTriangle // TableForm
```

Out[4]//TableForm=

1												
1	2											
1	4	2										
1	6	8	2									
1	8	18	12	2								
1	10	32	38	16	2							
1	12	50	88	66	20	2						
1	14	72	170	192	102	24	2					
1	16	98	292	450	360	146	28	2				
1	18	128	462	912	1002	608	198	32	2			
1	20	162	688	1666	2364	1970	952	258	36	2		
1	22	200	978	2816	4942	5336	3530	1408	326	40	2	
1	24	242	1340	4482	9424	12642	10836	5890	1992	402	44	2

Many sequences derived from the triangle turn out to have other combinatorial interpretations.
For instance, columns from the table give sequences counting integer lattice points of fixed L1-norm, given by Conway and Sloane.

The fourth column is Sloane's A035597:

```
In[5]:= AfTriangle[[Range[4, 12], 4]]

Out[5]= {2, 12, 38, 88, 170, 292, 462, 688, 978}
```

The first interesting diagonal in the table turns out to be Sloane's A005899:

```
In[6]:= AfTriangle[[Range[3, 12], -3]]

Out[6]= {1, 6, 18, 38, 66, 102, 146, 198, 258, 326}
```

Totalling the rows gives Sloane's A001333:

```
In[7]:= Total /@ AfTriangle

Out[7]= {1, 3, 7, 17, 41, 99, 239, 577, 1393, 3363, 8119, 19601, 47321}
```

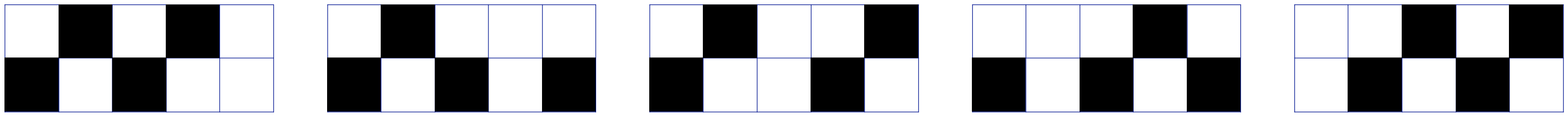
The entire triangle, read row-by-row, can be found as Sloane's A035607.

Since we have a computer, we might as well generate pictures (at least, that's the moral I've learned from modern calculus textbooks).
We will say two selections from the 2xN grid have the same class if their projections onto the x-axis agree.
A given projection onto the x-axis has 2^r lifts to selections on the grid, where r is the number of connected components in the projection. For purposes of illustrating the possibilities, one selection from each class seems to be sufficient.

■ Code for enumerating the possible selections and illustrating them

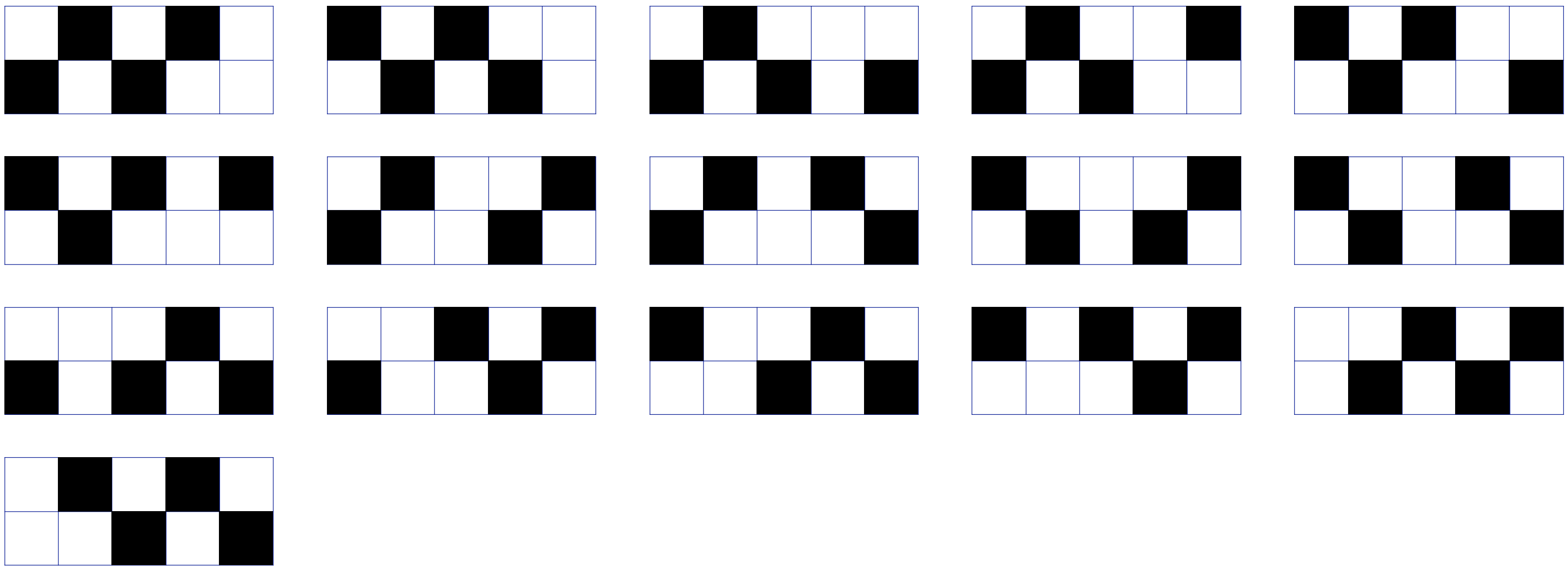
PrettyPic[n,k] will give us a picture of one selection in each class (for the problem of choosing k squares from a 2xN grid):

```
In[16]:= Show@PrettyPic[5, 4];
```



PrettyBigPic will give us a picture of all the selections (for the problem of choosing k squares from a 2xN grid):

```
In[17]:= Show@PrettyBigPic[5, 4];
         Af[5, 4]
```



```
Out[18]= 16
```

The actual sets of squares can be generated by AllLifts. The first picture in the second row above, for instance:

```
In[19]:= selections = AllLifts[5, 4];
         Dimensions@selections
         selections[[6]]

Out[20]= {16, 4, 2}

Out[21]= {{1, 2}, {2, 1}, {3, 2}, {5, 2}}
```