Square-wave self-modulation in diode lasers with polarization-rotated optical feedback

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The square-wave response of edge-emitting diode lasers subject to a delayed polarization-rotated optical feedback is studied in detail. Specifically, the polarization state of the feedback is rotated such that the natural laser mode is coupled into the orthogonal, unsupported mode. Square-wave self-modulated polarization intensities oscillating in antiphase are observed experimentally. We find numerically that these oscillations naturally appear for a broad range of values of parameters, provided that the feedback is sufficiently strong and the differential losses in the normally unsupported polarization mode are small. We then investigate the laser equations analytically and find that the square-wave oscillations are the result of a bifurcation phenomenon. © 2006 Optical Society of America

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Optical feedback from an external reflector is a well-known technique for inducing dynamic effects in semiconductor lasers. The behavior that results depends on many factors, such as the laser architecture and operating conditions, the external cavity length and configuration, and the strength of the feedback. Recently, the effect of polarization-rotated optical feedback (PROF) has raised considerable interest. In this setup, the polarization state of the optical feedback is orthogonal to the laser’s natural operating mode. Such systems are of interest from a variety of standpoints. First, their dynamic properties can be investigated in more detail than for conventional optical feedback, and they also apply for purely optoelectronic feedback systems. Second, the effect of PROF on vertical-cavity surface-emitting lasers has been examined in several laboratories. The feedback typically gives rise to switching between the dominant polarization mode and the orthogonal mode that is nearly degenerate due to the cylindrical symmetry of such devices. Polarization self-modulation has a variety of useful applications stemming from the production of optical pulses at multi-GHz repetition rates without the need for high-speed electronics. But edge-emitting lasers can also be made to exhibit pulse trains generated by polarization self-modulation, although the losses in the orthogonal mode are typically higher than in vertical-cavity surface-emitting lasers.

If a quarter-wave plate is used to create PROF, it allows mutual coupling between the polarization modes as well as multiple cavity round-trips. In this Letter, we use a Faraday rotator to avoid multiple round-trip reflections and allow only unidirectional coupling between the natural horizontal polarization (TE) mode and the unsupported orthogonal (TM) mode. This configuration also simplifies the analysis since the TM mode, even when activated by feedback, cannot influence the TE mode directly via optical injection. Recent studies of the dynamics of this system and its synchronization properties demonstrate the need to consider the intensities of both modes as independent variables. Rate equations have been used and reproduce the experimental observations. The objective of this Letter is to show analytically that square-wave output naturally appears in PROF systems.

The experimental apparatus is described in Fig. 1. We use an index-guided SDL-5401 laser diode (LD) with wavelength $\lambda = 817.9$ nm and current threshold of 18.48 mA. It is stabilized in temperature and pumped at 28.02 mA, which produces single-mode
description of the TM mode with \( k > 0 \) defined as the ratio of the TM and TE gain coefficients and the differential losses, respectively. The last term in Eq. (2) models the reinjection of the delayed, rotated TE field. Finally, Eq. (3) describes the carrier density, which depends on the intensities of both fields. All parameters are dimensionless. In addition to \( k \) and \( \beta \), the laser parameters include linewidth enhancement factor \( \alpha \), the ratio \( T \) of the carrier lifetime to the cavity lifetime, and the pump parameter above threshold, \( P \). The feedback is controlled by its strength \( \eta \) and its delay \( \tau \). The range of parameters for the square-wave regimes appears to be fairly broad and requires only small values of \( \beta \). Laser parameters representative of a SDL diode laser are \( T = 150, \alpha = 2, \) and \( P = 0.5 \). Equations (1)–(3) have been studied systematically \(^\text{16} \) for different values of \( k = 0.3–1, \beta = 0.03–0.09, \eta = 0.1–0.4, \) and \( \tau = 10^2–10^4 \). By gradually increasing \( \eta \), the nonzero intensity steady state transfers its stability to sustained relaxation oscillations that are progressively modulated by a \( 2\tau \)-periodic square-wave. Above a critical value of \( \eta \), the square-wave oscillations dominate and exhibit only decaying relaxation oscillations at the beginning of each square wave, as shown in Fig. 3. In simulations, we find that square-wave oscillations with more than two plateaus and periods longer than \( 2\tau \) are possible. However, the third plateau is unstable, is not seen experimentally, and does not enter into the analysis to follow. In simulations, a small amount of noise of amplitude \( 10^{-6} \) is introduced in the right-hand sides of Eqs. (1) and (2) to prevent the trajectory from residing near the unstable plateau.

After introducing \( E_j = A_j \exp(i \phi_j) \) \((j=1, 2)\) and the new time \( s = t/\tau \) into Eqs. (1)–(3), we neglect all \( \tau^{-1} \) small terms multiplying the time derivatives. The resulting algebraic equations are

\[
NA_1 = 0, \quad (4)
\]

\[
k(N-\beta)A_2 + \eta A_1(s-1)\cos(\Phi) = 0, \quad (5)
\]

\[
\alpha[N(s-1) - k(N-\beta)]A_2 - \eta A_1(s-1)\sin(\Phi) = 0, \quad (6)
\]

\[
P - N - (1 + 2N)(A_1^2 + A_2^2) = 0, \quad (7)
\]

Equation (1) describes the TE mode, which lases naturally since it has the lowest losses. Equation (2) describes the TM mode with \( k > 0 \) defined as the ratio of the TM and TE gain coefficients and the differential losses, respectively. The last term in Eq. (2) models the reinjection of the delayed, rotated TE field. Finally, Eq. (3) describes the carrier density, which depends on the intensities of both fields. All parameters are dimensionless. In addition to \( k \) and \( \beta \), the laser parameters include linewidth enhancement factor \( \alpha \), the ratio \( T \) of the carrier lifetime to the cavity lifetime, and the pump parameter above threshold, \( P \). The feedback is controlled by its strength \( \eta \) and its delay \( \tau \). The range of parameters for the square-wave regimes appears to be fairly broad and requires only small values of \( \beta \). Laser parameters representative of a SDL diode laser are \( T = 150, \alpha = 2, \) and \( P = 0.5 \). Equations (1)–(3) have been studied systematically \(^\text{16} \) for different values of \( k = 0.3–1, \beta = 0.03–0.09, \eta = 0.1–0.4, \) and \( \tau = 10^2–10^4 \). By gradually increasing \( \eta \), the nonzero intensity steady state transfers its stability to sustained relaxation oscillations that are progressively modulated by a \( 2\tau \)-periodic square-wave. Above a critical value of \( \eta \), the square-wave oscillations dominate and exhibit only decaying relaxation oscillations at the beginning of each square wave, as shown in Fig. 3. In simulations, we find that square-wave oscillations with more than two plateaus and periods longer than \( 2\tau \) are possible. However, the third plateau is unstable, is not seen experimentally, and does not enter into the analysis to follow. In simulations, a small amount of noise of amplitude \( 10^{-6} \) is introduced in the right-hand sides of Eqs. (1) and (2) to prevent the trajectory from residing near the unstable plateau.

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Fig. 4. Bifurcation diagram of the square-wave self-modulated solutions. $P=0.5$, $a=2$, $\eta=0.35$, and $k=1/3$. The figure represents the difference between the values of $|E_2|$ and $|E_1|$ for each plateau of the square wave as a function of $\beta$. If $\beta > \beta_c$, chaotic oscillations appear.

where $\Phi = \phi_1(s-1) - \phi_2$. Equations (4)–(7) relate the values of the dependent variables at time $s$ to their values at time $s-1$. Equation (4) is satisfied if either $N=0$ or $A_1=0$. We anticipate that the square-wave solution exhibits two successive stages characterized by (1) $N=0$ and $A_1(s-1)=0$ and (2) $A_1=0$ and $N(s-1)=0$. For stage (1), we then find from the remaining equations that $A_2=0$ and $A_1=\sqrt{P}$. For stage (2) now supplemented by $A_1(s-1)=1$, we find from Eqs. (5) and (6) that $\tan(\Phi)=\alpha$ and

$$k(N-\beta)A_2 + \eta \sqrt{\frac{P}{1+\alpha^2}} = 0. \quad (8)$$

Finally, from Eq. (7) with $A_1=0$, we determine $A_2$ as

$$A_2^2 = \frac{P-N}{1+2N}. \quad (9)$$

Equations (8) and (9) lead to the bifurcation diagram of the square-wave solutions. In Fig. 4, we represent the difference between the values of the two polarization plateaus, i.e., $A_2-A_1$, where $A_2$ is determined from Eq. (9) and $A_1=\sqrt{P}$. Using $\beta$ as the bifurcation parameter, the square-wave solution exists if $N \leq 0$, i.e., if

$$\beta \leq \beta_c = \frac{\eta}{k \sqrt{1+\alpha^2}}. \quad (10)$$

If $\eta=0.35$, $k=1/3$, and $a=2$, then $\beta_c = 0.47$. Using these values of $\eta$, $k$, $\alpha$, $P=0.5$, $T=150$, and $\tau=10^3$, we have verified numerically from Eqs. (1)–(3) that a stable square-wave regime exists at $\beta=0.46$ but disappears at $\beta=0.47$ into chaotic oscillations as were found previously in this system. 

For a given laser with fixed differential losses, Eq. (10) provides a condition on the minimum feedback at which square waves are possible. This agrees with numerical findings that require a sufficiently strong $\eta$, since higher $\eta$ implies a higher $\beta_c$. In both the experiments and the numerical simulations, the square-wave oscillations are supplied by decaying relaxation oscillations. They can be studied analytically by investigating the stability of the plateaus with respect to the fast time scale of the relaxation oscillation. The results of this analysis will be presented elsewhere.

In summary, we have found experimentally and theoretically that stable square-wave self-modulated oscillations of a period close to twice the round-trip time naturally appear for edge-emitting diode lasers subject to PROF. The main condition is that the differential losses in the TM mode are small so the two polarization modes may behave independently. In terms of this parameter, we then showed analytically that there exists a bifurcation point for the two main plateaus of square-wave oscillations above which a square-wave regime is no longer possible.

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