Identity synchronization in diode lasers with unidirectional feedback and injection of rotated optical fields

David W. Sukow,1,* Athanasios Gavrielides,2 Taylor McLachlan,1 Guinevere Burner,1 Jake Amonette,1 and John Miller1
1Department of Physics and Engineering, Washington and Lee University, 204 West Washington Street, Lexington, Virginia 24450, USA
2Air Force Research Laboratory, Directed Energy Directorate AFRL/DELO, 3550 Aberdeen Avenue SE, Kirtland AFB, New Mexico 87117, USA

(Received 3 May 2006; published 16 August 2006)

Identity synchronization is observed experimentally and numerically in the chaotic dynamics of a system of two unidirectionally coupled semiconductor lasers. The transmitter and receiver lasers are subjected to polarization-rotated optical feedback and injection, respectively. Numerical and analytical results show that identity synchronization requires parameter matching through a relationship between the injection and feedback strengths, and linewidth enhancement factors of the lasers. Inverse synchronization is also observed experimentally.

DOI: 10.1103/PhysRevA.74.023812

PACS number(s): 42.65.Sf, 42.55.Px, 05.45.Xt

I. INTRODUCTION

Chaos synchronization in semiconductor lasers is a subject of much interest because of its applications to optical communications, and is also studied as a fundamental property of coupled, time-delay systems. Such phenomena have been found for various configurations and laser devices, including all-optical [1–5] and optoelectronic [6,7] feedback scenarios, including unidirectional and mutual coupling. In addition, in several such systems, research has shown that different forms of synchronization can exist. For example, in systems for which the transmitter is made chaotic using some form of delayed feedback, identity synchronization is found when a sufficiently similar receiver laser is injected unidirectionally under conditions that the feedback and injection rates are comparable. However, if the injection rate is much larger than the feedback rate, driving synchronization is observed, where the receiver is synchronized directly to the delayed injected signal, rather than to the full chaotic response of the transmitter [8–10]. The simplest way to distinguish which form of synchronization is present is by the time lag between the two lasers, which differs between the two solutions by the external cavity roundtrip time.

One particular system of recent interest is unidirectionally coupled lasers, for which both the feedback and injection fields are rotated in polarization such that they are orthogonal relative to the natural emission modes of the laser. Previous experiments have examined several configurations for the chaotic transmitter in such systems [11–13]. Such feedback and injection are sometimes called incoherent because the phases of the injected signals are presumed not to influence the dynamics, and indeed, the square-law optoelectronic feedback system is dynamically equivalent to this system when viewed by a simple model involving only the natural polarization mode of the lasers [14,15]. However, whereas the optoelectronic system has only been observed to display identity synchronization [16,17], the polarization-rotated optical system has only been observed to display driving synchronization in previous experiments [18,19], a result suggesting that a full two-field model is required to fully describe the synchronization phenomena in this system [12,20].

Recent numerical studies with a two-field model have demonstrated that both driving and identity synchronization should be observed in the polarization-rotated optical system [21]. In this paper, we report an experimental observation of the identity synchronization state. Numerical and analytical studies demonstrate that matched injection and feedback rates are important, as are matched linewidth enhancement factors, but differences in one can be compensated by the other. Under proper operating conditions, agreement between experimental and numerical findings is very good. We also make a preliminary experimental report of inverse synchronization occurring in the system as well.

We structure this paper by first presenting the mathematical model, followed by the synchronization solutions and their conditions. Subsequently, the experimental apparatus is described, followed by a presentation and discussion of the identity and inverse synchronization data.

II. MATHEMATICAL MODEL

To investigate the complete or identity synchronization in the polarization-rotated optical feedback and unidirectional injection problem, we will use the two-polarization description [12,19] expressed in dimensionless form [19]. This model includes two complex electric fields $E_1$ and $E_2$ with orthogonal linear polarizations, and a carrier density $N$ for each of the two lasers. In the transmitter (denoted by the superscript $t$), the natural or the lasing polarization is rotated to the orthogonal polarization and re-injected into the diode laser after a delay $\tau$ in the external cavity. Thus the model for the transmitter necessarily incorporates a description not only of the natural horizontal TE polarization $E_1^t$ but also an equation that describes the orthogonal or vertical TM polarization $E_2^t$. Similarly, for the receiver (denoted by the superscript $r$) the injected field in the vertical polarization equation $E_2^r$ is proportional to the rotated horizontal polarization of the

*Electronic address: sukowd@wlu.edu
transmitter after a delay $\tau_c$ equal to the time of flight from the transmitter to the receiver. Thus this treatment is restricted to the case of the injection or open loop synchronization only. The injection term is not directly coupled into the receiver’s horizontal polarization $E_1^r$ but is mediated through the carriers $N^r$.

In general, the TE and TM modes will have different gains, but for simplicity in this presentation we will assume that they are the same for both polarizations. The losses $\beta$ distinguish the dominant lasing mode as the horizontal polarization. A more detailed model that includes the effects of differential gain between the two modes is discussed in Ref. [22]. There are several combinations of differential gains and losses that lead to synchronization, so we have picked the simplest without loss of generality. The equations are

$$\frac{dE^r_t}{dt} = (1 + ia)N^r E^r_t,$$
(1)

$$\frac{dE^e_r}{dt} = (1 + ia)(N^e - \beta)E^e_r + \eta e^{-i(\omega_1 t + \phi_e)} E^r_1(t - \tau),$$
(2)

$$T \frac{dN^e}{dt} = P - N^e - (1 + 2N^e)[|E^e_t|^2 + |E^e_r|^2],$$
(3)

$$\frac{dE^r_t}{dt} = (1 + ia)N^r E^r_t,$$
(4)

$$\frac{dE^e_t}{dt} = -i\Omega E^e_t + (1 + ia)(N^e - \beta)E^e_t + \eta e^{-i(\omega_1 t + \phi_e)} E^r_1(t - \tau),$$
(5)

$$T \frac{dN^e}{dt} = P - N^e - (1 + 2N^e)[|E^e_t|^2 + |E^e_r|^2].$$
(6)

In these equations, the key parameters are the ratio of the carrier density and cavity lifetimes $T$, the linewidth enhancement factor $a$, the pump parameter above threshold $P$, and the differential losses $\beta$. The feedback and injection parameters $\eta$ and $\eta^\prime$, respectively, can be controlled independently.

The first three equations describe the transmitter or master laser, and all the appropriate parameters and dynamical variables are denoted by the superscript $t$. We have assumed that the frequencies of both fields in the transmitter are equal, that is, $\omega^t_1 = \omega^t_2$, because the re.injected polarization-rotated feedback that activates $E^r_2$ encounters no elements in the external cavity that could induce a frequency shift. However, this is not the case in the receiver or slave laser. Its horizontal polarization $E^r_1$ lases at a natural frequency $\omega^r_1$ but the vertical polarization $E^e_2$ will be at the same frequency as the injected field, that is, $\omega^e_2 = \omega^t_1 = \omega^t_2$. This effect is included in Eq. (5) by the term $\Omega = \omega^r_1 - \omega^e_2$. Indeed, we have found experimentally there is a detuning between the injected field from the transmitter and the field of the receiver. In addition, this detuning can be controlled by adjusting the temperature of the receiver by $\pm 5^\circ C$. This corresponds to as much as $\pm 1$ nm and it can affect the strength of the injection [12,19]. A quick estimate shows that taking $\Delta\lambda = 1$ nm for the wavelength separation between the two polarizations and for $\gamma_p = 5 \times 10^{11}$ s$^{-1}$ for the cavity lifetime at $\lambda = 1 \mu m$ shows that $\Omega \sim 1$. We assume for analytical purposes that all the other parameters that characterize the lasers are identical, although we shall relax this restriction later for numerical investigations of parameter mismatch. Additionally, we assume for simplicity that the two lasers operate at the same optical frequency, that is, $\Omega = 0$. It can be shown easily that any phases can be eliminated completely by rescaling, for example, $E^r_1$ and $E^e_2$ by the appropriate phase factors that appear in Eqs. (2) and (5). Indeed, numerical computations show that the dynamics and the synchronization of the transmitter and receiver are unaffected. Therefore, we no longer have to take into account any such phase factors in the synchronization conditions.

We can easily deduce the necessary conditions for identity synchronization by an examination of Eqs. (1)–(6):

$$\eta^\prime = \eta^\prime$$
(7)

and

$$E^r_1(t) = E^r_1(t - \tau_c + \tau),$$
(8)

$$E^e_2(t) = E^e_2(t - \tau_c + \tau).$$
(9)

Equations (8) and (9) indicate the identity synchronization between transmitter and receiver with a lag or an anticipative synchronization depending on the magnitude of the two delays. In addition, under the conditions discussed in detail and the derivations in the Appendix of Ref. [19] we obtain the approximations

$$E^r_1(t) \sim E^r_1(t - \tau_c),$$
(10)

$$E^e_2(t) \sim E^e_2(t - \tau_c).$$
(11)

In light of the discussions in Refs. [23,24] and based on the identity synchronization of Eq. (8), then Eqs. (10) and (11) should be regarded as expressing that these fields are in generalized synchronization. Indeed, $E^r_1(t)$ and $E^e_2(t - \tau_c + \tau)$ can be identified as auxiliary response systems which are driven by the output of an autonomous drive system $E^r_1(t)$ delayed by either $\tau$ or $\tau_c$.

We can illustrate these conclusions in a series of numerical calculations when the transmitter and the receiver are in identity synchronization. We have taken the two lasers to have identical parameters including the delays. For demonstration purposes the delays are assumed to be given by $\tau = \tau_c = 1400$. The rest of the parameters are given by $T = 1000.0$, $a = 2.0$, $P = 0.5$, and $\beta = 0.35$. We also assume that both lasers are operating at the same optical frequencies in all polarization modes. The value of the feedback $\eta^\prime$ is selected so that the transmitter laser is operating chaotically, and the injection strength $\eta$ is set equal to it, $\eta = \eta^\prime = 0.38$. The time has been normalized to the dimensionless relaxation frequency given by $\omega = \sqrt{2P/T}$. The normalized feedback and injection delays then are given by $\vartheta = \vartheta_e = \omega \tau = 44.27$. Figure 1(a) shows the time series of the total intensity of the transmitter laser given by $I_p = |E^r_1|^2 + |E^e_2|^2$, and Fig. 1(b) shows the transmitter intensity versus the receiver
intensity. It is obvious that the two lasers are synchronized identically with a correlation of $C = 1.0$ with a relative delay equal to zero as predicted from Eqs. (8) and (9). A slight thickening of the diagonal line indicates the small amount of noise injected into the calculation. All quantities displayed are dimensionless in this and other figures displaying numerically calculated data.

In Fig. 2 we investigate the time-shifted correlation between the polarization components of the transmitter and receiver. The correlations in the figures are arranged according to the equations that define the synchronization manifold Eqs. (8) and (9) and the approximate or generalized synchronization in Eqs. (10) and (11). In particular, Figs. 2(a) and 2(b) show that the correlation between the horizontal polarizations of the receiver and the transmitter is equal to 1.0, as is the correlation of the vertical polarizations of the two devices. The relative delay $\delta \tau$ at maximum correlation is equal to zero, since the feedback delay of the transmitter is equal to the injection delay to the receiver, as verified by the equations. On the other hand, Figs. 2(c) and 2(d) verify Eqs. (10) and (11) with a relative lag delay of $\delta \tau \approx 44.22$ approximately equal to $\vartheta_1$, and to $\vartheta$ with a correlation of 0.94 for both cases.

To further verify the theoretical predictions we have numerically computed the synchronization for unequal paths $\tau = 1400$ and $\tau_c = 1580$ keeping the rest of the parameters unchanged for the two lasers as in the previous calculation. The results are shown in Table I and confirm very accurately the predictions of Eqs. (8)–(11). The identities indicated by the first two equations are satisfied with a correlation of practically 1.0, and the delays at maximum correlation compare very well to within the accuracy of the calculation. Additionally, the last two equations are satisfied to a high accuracy both for the degree of correlation and also to the relative delay.

Finally, we have also computed the maximum degree of correlation for identity synchronization for diode lasers with mismatch in one of the parameters. In Fig. 3 we plot the maximum correlation between the intensities of transmitter and receiver for a mismatch between the feedback of the transmitter and the strength of the injection in the receiver. The figure depicts the degree of synchronization as the strength of the feedback in the transmitter is held constant to a value of $\eta = 0.38$, while $\eta$ is varied. Note that for positive and negative mismatches of injection the maximum correlation degrades significantly, but nevertheless for all displayed data points the relative delay is preserved at the expected value of $\tau_c - \tau = 180$. However, outside the last points in this graph the computed maximum correlation values can be relatively high, but the relative delays become close to the injection delay $\tau_c$ due to the optical path length from transmitter to receiver, indicating driving synchronization.

Similarly, Fig. 4 shows the maximum correlation as a function of mismatch of the linewidth enhancement factors of the transmitter and receiver. We keep the linewidth
enhancement factor of the transmitter constant at $\alpha^t=2.0$ and vary its value in the receiver $\alpha'$ to compute the effects on the synchronization correlation. As in Fig. 3, the relative delay is computed at maximum correlation and for all displayed data points it is equal to $\tau_r - \tau = 180$. Again, outside the computation points the synchronization correlation can remain relatively high but with the relative delays close to driving lag $\tau_c$.

The similarity between Figs. 3 and 4 is not purely coincidental, but is due to the fact that the factor $\eta/\beta \sqrt{1 + \alpha^2}$ controls the feedback of the transmitter and the injection of the receiver. Therefore variation in $\eta$ can actually be compensated by the differences in the linewidth enhancement factor or for that matter in variation in the relative cavity losses of the modes. This also explains how identity synchronization was achieved experimentally, as will be shown in the next section, even though the feedback and injection rates were different. As an example we show a calculation for which the injection strength is $\eta=0.473$ compared to the feedback of $\eta=0.38$, the same as in all the previous calculations. The linewidth enhancement factor of the receiver is set to $\alpha^r=2.6$ compared to $\alpha^t=2.0$. Note that $\eta/\beta \sqrt{1 + \alpha^2}=0.486$ for both devices and that the receiver parameters fall quite outside the region over which identity synchronization is achieved if either of the parameters were used individually as shown in Figs. 3 and 4. Nevertheless, we find that the maximum correlation of the intensities between the receiver and transmitter is 0.98 and with a relative delay of $\delta_r - \delta = 5.64$. As a visual aid we show the intensities of receiver and transmitter plotted against each other in Fig. 5.

III. EXPERIMENT

A. Apparatus

The apparatus for our experimental studies of identity synchronization consists of a transmitter laser in a ring-cavity configuration which provides polarization-rotated optical feedback to induce chaos. The receiver is injected unidirectionally with an optical signal from the transmitter that is essentially the same as the feedback. This configuration has the advantage over a linear cavity that the feedback and injection strengths can be varied independently with ease. The system is illustrated schematically in Fig. 6.

Both the transmitter (LD1) and receiver (LD2) lasers are of the same model (SDL 5401-G1). Chaos is induced in the transmitter laser using delayed optical feedback from a ring cavity as follows. The horizontally polarized beam emerging from LD1 is collimated by a lens (CL) with numerical aperture of 0.47, passes through a nonpolarizing plate beamsplitter (BS, $R=30\%$) which provides a detection beam, and then transmits unimpeded through a polarizing beamsplitter cube.
(PBS) whose transmission axis is parallel to the horizontal beam. It then passes through a sequence of two Faraday isolators (ISO), each of which rotate the beam’s polarization by 45°, resulting in a vertically polarized beam at this point. This beam then strikes a partial mirror (PM, R = 30%) which separates it into two portions, one forming the feedback loop and another forming the injection beam. The feedback beam passes through a rotatable linear polarizer (POL) which controls the feedback strength, then strikes for a second time the PBS. Since the beam now has a vertical polarization state, it reflects from the PBS and is re-injected into LD1 after again traversing the plate beamsplitter and the collimating lens. The total ring cavity length is 130 cm, yielding a roundtrip time $\tau = 4.33$ ns. Note that the ring cavity design assures that only the horizontal mode couples into the vertical mode and that no multiple-reflection roundtrips are sustained, since any vertically polarized light will circulate through the cavity in the opposite direction and thus be effectively extinguished when reaching the Faraday isolators (−40 dB reverse transmission per isolator).

The portion of LD1’s output that is transmitted through the partial mirror (PM) after passing through the two Faraday isolators becomes the injection beam for synchronizing LD2. As noted above, the beam is vertically polarized at this point in the system. It then strikes two high-reflectivity steering mirrors (HR), is attenuated by a variable, rotatable neutral density filter (ATT), transmits through a plate beamsplitter (BS, $R = 30\%$), and couples into LD2 via a collimating lens (CL) identical to that used for LD1. The injection path length is 146 cm for an injection time of $\tau = 4.87$ ns. Although there is a counterpropagating beam emitted from LD2, the dual isolators again ensure unidirectional coupling.

To characterize the optical spectra of both lasers in this configuration, we use a high-finesse nonconfocal scanning interferometer (Newport SR-240C, free spectral range 1100 GHz, finesse $> 13,000$). With feedback and injection active, both polarization modes of both lasers still exhibit a single longitudinal mode, consistent with the mathematical model’s use of only two complex fields $E_1$ and $E_2$ for each laser. Furthermore, in the transmitter, the optical frequencies of both fields are the same: $\omega_{1f} = \omega_{2f}$. However, the frequencies of the injected receiver’s two fields are observed to differ from each other: $\omega_{1r} \neq \omega_{2r}$. We also note that both receiver frequencies $\omega_{1r}$ and $\omega_{2r}$ differ from the natural frequency of the solitary receiver in the absence of injection. Further details on the interesting frequency effects in this system will be documented elsewhere.

For synchronization measurements, the intensity dynamics of both LD1 and LD2 are sampled immediately after they emerge from their respective collimating lenses, by use of the plate beamsplitters (BSs), which direct 30% of the incident light to the detection arms. For both lasers, linear polarizers are inserted into the beam path to allow polarization-resolved detection, as the transmitted beams strike identical ac-coupled photodetectors (PD1 and PD2) with detection bandwidths of 8.75 GHz (Hamamatsu C4258-01). Neutral density filters (not shown) attenuate both beams to limit the power incident on the detectors. The detected signals are amplified with ac wideband (10 kHz–12 GHz) microwave amplifiers (AMP) with 23 dB gain, and then are connected with identical high-performance coaxial cables to either an rf spectrum analyzer (Agilent E4405B, 13.2 GHz BW) or a high speed digitizing oscilloscope (LeCroy 8600, 6 GHz analog bandwidth and 50 ps sampling interval). The detector and transmission cable responses are identical within our ability to measure them. The detection path lengths from LD1 to PD1 and from LD2 to PD2 are 45 and 71 cm, respectively, which introduce detection lags of 1.50 and 2.37 ns. All such lags which are artifacts originating from detection apparatus are accounted for when determining the actual synchronization lag between the chaotic lasers. They have no physical relevance to the dynamics and therefore will be eliminated from all subsequent discussion and data presentation, to allow a clearer focus on the key synchronization time lag issues, which are critical to determine the synchronization solution observed.

To confirm synchronization experimentally, we rely primarily on time series data. We use the fast oscilloscope to acquire simultaneous data sets from specific pairs of linear polarizations of the transmitter and receiver lasers. The captured time series have lengths of at least 4000 points with a 50 ps sampling interval for a minimum total duration of 200 ns. Cross-correlation analysis of this data reveals both the degree of synchronization, as well as the time lag between transmitter and receiver for which the highest correlation occurs. This is important since the lag is the primary way to distinguish between the driving or identity synchronization solutions. Also, there are several lags at which the correlation function will have a local peak because the signal has oscillatory components at a number of frequencies, but for chaotic time series these diminish relatively quickly.

The cross-correlation function $S(\Delta t)$ relates the oscillations in the pair of chaotic waveforms using a time-shifted function

$$S(\Delta t) = \frac{\langle P_1(t)P_2(t-\Delta t)\rangle}{\langle P_1^2(t)\rangle^{1/2} \langle P_2^2(t)\rangle^{1/2}},$$

(12)

where $P_1(t)$ and $P_2(t)$ are the ac components of the transmitter and receiver powers, respectively, and $\Delta t$ is the shift between the two time series at which the cross correlation is calculated. The time $\Delta t$ at which the largest absolute value of $S(\Delta t)$ occurs corresponds to the synchronization lag. $S(\Delta t)$ is normalized such that it ranges from +1 (identical signals) to −1 (identical but opposite in sign). Unrelated signals should have a value approaching zero.

**B. Identity synchronization**

For all experimental results presented for identity synchronization, the transmitter LD1 operates at a stabilized temperature of 20.02 °C resulting in a wavelength of $\lambda_1=816.9$ nm. Its operating drive current is 36.03 mA, roughly twice its solitary threshold of 17.63 mA. The receiver LD2 runs at 18.91 °C and $\lambda_2=817.8$ nm, operates at 34.84 mA and has a solitary threshold of 16.94 mA. For the data shown, the external cavity roundtrip transmission is $T_{ext}=9.8\%$ and the transmission through the injection path is $T_{inj}=8.3\%$, both quantities expressed as a fraction of LD1’s total output power.
time lags the horizontal mode by the external cavity roundtrip in the previous theoretical section, the vertical mode of LD1 transmitter dynamics only has shown, and as was reviewed the individual lasers themselves. As past research on the a time lag between the horizontal and vertical modes of synchronization lags.

An additional subtlety must be recalled, that there is a time lag between the horizontal and vertical modes of the individual lasers themselves. As past research on the transmitter dynamics only has shown, and as was reviewed in the previous theoretical section, the vertical mode of LD1 lags the horizontal mode by the external cavity roundtrip time $\tau$. This is a natural consequence of the suppressed mode being activated by the polarization-rotated feedback. Therefore, we expect the lag between LD1 and LD2 to be equal to the identity synchronization lag of $\tau_{r}-\tau$ only when pairs of like polarizations are measured. However, if we compare the horizontal mode of the transmitter to the vertical mode of the receiver, an additional lag of $\tau$ is predicted (which makes it equal to what one would normally expect for driving synchronization $\tau_{r}$).

Figure 7 shows the cross-correlation functions $S(\Delta t)$ calculated for two pairs of time series under the perfect synchronization solution. In Fig. 7(a), like polarizations for the transmitter and receiver are shown (both vertical). Figure 7(b) shows a close-up view of the same data. The largest peak value, 0.732, appears at a time shift $\Delta t$ of 0.38 ns. This measured shift agrees with the predicted value for identity synchronization approximately within the detection bandwidth of the system, and moreover is distinguished easily from the value associated with the driving solution. The nearest adjacent peaks of $S(\Delta t)$ are clearly smaller, and the correlation becomes small within a short time shift as is expected for a chaotic signal. The third graph, Fig. 7(c), shows $S(\Delta t)$ for unlike polarizations, the horizontal mode of the transmitter and vertical mode of the receiver, with Fig. 7(d) showing the region near the correlation maximum. Now the largest peak value 0.785 is located at $\Delta t=4.98$ ns. Thus the measured delay is again in agreement with the predicted value of 4.87 ns for unlike polarizations. As explained above, the larger expected delay in this case is due to the additional $\tau$ delay between horizontal and vertical modes within the same laser.

The time series data from which the correlation functions were calculated are displayed in Fig. 8. For ease of visual comparison, we scale and shift the axes for each time series. All waves are scaled vertically by subtracting the mean and normalizing the standard deviation to unity, and the lagging wave of each pair is shifted horizontally by the $\Delta t$ associated with the maximum cross correlation.

Figure 8(a) shows the synchronization between the vertically polarized modes of the transmitter and receiver. The transmitter and receiver lasers are represented by the thin black and thick gray curves, respectively. The data appear very similar, supported by the maximum cross-correlation coefficient of $S=0.732$. Figure 8(b) shows the case for the horizontal mode of the transmitter and the vertical mode of the receiver. The signals and synchronization are similar, but in this case the receiver has been shifted by $\Delta t=4.98$ ns.
As an alternate visualization of the data in Fig. 8, in Fig. 9 the shifted time series are plotted against each other. As is expected for synchronized waves, the points lie along the diagonal, and the thickness of the ellipses is a measure of the quality of the synchronization.

C. Inverse synchronization

Under slightly different operating conditions, we observe a different synchronization phenomenon: inverse synchronization, also called antisynchronization. This form of synchronization is characterized by two waves possessing a correlation coefficient that is negative at the time lag when the maximum magnitude is observed. Rather than being unrelated, therefore, the waves act as mirror images of each other, one peaking while the other troughs, with oppositely directed zero crossings in between. Recall that our experimental system provides only ac detection, and therefore the negative optical powers observed are in fact an expression of the optical power below the mean value. Inverse synchronization has been studied in simple delay models [25], has been demonstrated experimentally in the diode laser with an external cavity in open loop synchronization [26,27], but a general understanding of the phenomenon is not yet complete. We present here a preliminary experimental observation of this phenomenon in the polarization-rotated feedback and injection system.

For all results in this subsection, the transmitter LD1 operates at 20.02 °C and $\lambda_1 = 817.0 \text{ nm}$. Its operating drive current is kept at 36.03 mA. The receiver LD2 runs at 18.10 °C and $\lambda_2 = 817.2 \text{ nm}$, and is pumped at 34.69 mA. The external cavity roundtrip transmission is $T_{\text{ext}} = 12.0\%$ and the transmission through the injection path is $T_{\text{inj}} = 5.4\%$, expressed as a fraction of LD1’s total output power.

Figure 10 shows a cross-correlation function $S(\Delta t)$ calculated for two pairs of time series captured as the system exhibits inverse synchronization. For these data, the natural, horizontal polarization mode of each laser is measured. In Fig. 10(a), the largest peak value $-0.84$ appears at a time shift of $5.08 \text{ ns}$, corresponding to the lag expected for driving, not identity, synchronization. The correlation dies off quickly, again as expected for a chaotic signal. The second graph, Fig. 10(b) is a closer view of the largest peak. It is clear that the negative peak is significantly larger than

![FIG. 9. Synchronization plot for identity synchronization. Graphs (a) and (b) display the same normalized intensity data as in Figs. 8(a) and 8(b), with the receiver again shifted forward by 0.38 and 4.98 ns, respectively.](image)

![FIG. 10. Cross-correlation functions for inverse synchronization. Graph (a) displays $S(\Delta t)$ over the full range. The correlation shows a distinct negative peak and the function falls off quickly away from the maximum. Graph (b) is a close-up view of the largest peak. The negative peak is significantly larger than the two positive peaks on either side, confirming inverse synchronization.](image)

![FIG. 11. Time series demonstrating inverse synchronization. Optical powers have been normalized and the receiver is shifted forward in time by 5.08 ns for visual clarity.](image)

![FIG. 12. Synchronization plot for inverse synchronization. The graph displays the same normalized intensity data as in Fig. 11, with the receiver again shifted forward by 5.08 ns. The slope of $-1$ indicates inverse synchronization.](image)
either of the two adjacent positive peaks, confirming the synchronization as inverse.

The time series data from which the correlation function in Fig. 10 was calculated are displayed in Fig. 11. We scale and shift the axes for each time series as was done before for identity synchronization, for ease of visual comparison. The transmitter and receiver lasers again are represented by the thin black and thick gray curves, respectively, and the two waves display an inverse relationship. However, inverse synchronization is somewhat more difficult to gauge by direct visual comparison of waves than is a positive correlation. Therefore we present an alternate visualization of the same data in Fig. 12, in which the two time series are plotted against each other with the appropriate time shift. In this case the points lie along a line with a computed slope of −1 within the standard deviation, indicating inverse synchronization. The scatter in the ellipse again indicates the quality of the antisynchronization.

In summary, we have presented a theoretical analysis, numerical simulations, and experimental evidence demonstrating the observation of identity synchronization in the system of unidirectionally coupled semiconductor lasers with optical feedback and injection of polarization-rotated fields. We find that the identity synchronization solution occupies a small region of parameter space, and only appears when the two lasers and their operating parameters are similar. However, mismatch in one parameter can be offset by a compensating mismatch in another. We also have reported preliminary observations of inverse synchronization in this system. Theoretical understanding of this phenomenon is not yet complete, and thus remains a topic for further study.

ACKNOWLEDGMENTS

This material is based upon work supported by the U.S. National Science Foundation under CAREER Grant No. 0239413.